Math 220 (section AD?) **Quiz 9** **Spring 2016**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) A town currently has a population of 4000 people. Suppose the town's population grows by $0.75t^2$ people per year where t is measured in years from now. What will the town's population be in 10 years?

population in 10 years = current population + the net change in population from $t=0$ to $t=10$

$$= 4000 + \int_0^{10} (\text{rate of change of population})\, dt$$

$$= 4000 + \int_0^{10} 0.75t^2\, dt$$

$$= 4000 + \left[0.75\left(\tfrac{1}{3}t^3\right)\right]_0^{10}$$

$$= 4000 + 0.25(1000) - 0.25(0)$$

$$= 4000 + 250 = 4250 \text{ people}$$

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_1^4 \frac{4\sqrt{x} - 5x}{x}\, dx = \int_1^4 \left(\frac{4x^{1/2}}{x} - \frac{5x}{x}\right) dx$$

$$= \int_1^4 \left(4x^{-1/2} - 5\right) dx$$

$$= \left[4 \cdot \frac{1}{1/2} x^{1/2} - 5x\right]_1^4$$

$$= \left[8\sqrt{x} - 5x\right]_1^4$$

$$= (8\sqrt{4} - 5 \cdot 4) - (8\sqrt{1} - 5 \cdot 1)$$

$$= (16 - 20) - (3)$$

$$= -7$$

3. (2 points each) Evaluate the following indefinite integrals.

(a) $\int \frac{10x^3 + 10x + 3}{x^2 + 1}\, dx$

$= \int \left(10x + \frac{3}{x^2+1}\right) dx$

$= 5x^2 + 3\arctan(x) + C$

$\frac{10x^3+10x+3}{x^2+1} = \frac{10x(x^2+1)+3}{x^2+1}$

$= 10x + \frac{3}{x^2+1}$

or use polynomial long division

$$\begin{array}{r} 10x \\ x^2+1 \overline{\smash{)}\,10x^3+10x+3} \\ \underline{10x^3+10x} \\ 3 \end{array}$$

Thus,

$\frac{10x^3+10x+3}{x^2+1} = 10x + \frac{3}{x^2+1}$

(b) $\int \frac{\cos(2x)}{\cos(x) - \sin(x)}\, dx$

$= \int \frac{\cos^2(x) - \sin^2(x)}{\cos(x) - \sin(x)} dx$

$= \int \frac{(\cos(x)-\sin(x))\cdot(\cos(x)+\sin(x))}{\cos(x)-\sin(x)} dx$

$= \int (\cos(x) + \sin(x))\, dx$

$= \sin(x) - \cos(x) + C$

4. (2 points) Suppose $g(x) = \int_{x^3}^{4} \frac{1}{t^6+1}\,dt$. Find $g'(x)$.

$$g(x) = -\int_4^{x^3} \frac{1}{t^6+1}\,dt$$

$$g'(x) = -\frac{1}{(x^3)^6+1} \cdot \frac{d}{dx}(x^3)$$

from part 1 in Fundamental Theorem of Calculus and the chain rule

$$g'(x) = -\frac{1}{x^{18}+1} \cdot 3x^2$$

$$g'(x) = \frac{-3x^2}{x^{18}+1}$$

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or more formally,

$$g = -\int_4^{x^3} \frac{1}{t^6+1}\,dt$$

Let $u = x^3$. Then $\frac{du}{dx} = 3x^2$

$$g = -\int_4^{u} \frac{1}{t^6+1}\,dt$$

By F.T.C. (part 1), $\frac{dg}{du} = -\frac{1}{u^6+1}$

$$g'(x) = \frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = -\frac{1}{u^6+1} \cdot 3x^2 \quad \text{(chain rule)}$$

$$= -\frac{1}{(x^3)^6+1} \cdot 3x^2$$

$$= -\frac{3x^2}{x^{18}+1}$$