Math 220 (section AD?) Quiz 5 Spring 2016

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Given $R(t) = \sin\left(t^4\right)$, find its second derivative $R''(t)$.

$R'(t) = \cos(t^4)\cdot\frac{d}{dt}(t^4)$

$R'(t) = \cos(t^4)\cdot 4t^3$

$R''(t) = \frac{d}{dt}(\cos(t^4))\cdot 4t^3 + \cos(t^4)\cdot\frac{d}{dt}(4t^3)$

$R''(t) = -\sin(t^4)\cdot\frac{d}{dt}(t^4)\cdot 4t^3 + \cos(t^4)\cdot 12t^2$

$R''(t) = -\sin(t^4)\cdot 4t^3\cdot 4t^3 + \cos(t^4)\cdot 12t^2$

$R''(t) = -16t^6\sin(t^4) + 12t^2\cos(t^4)$

2. (3 points) Compute $w'(x)$ given that $w(x) = \tan^5\left(\sqrt{x^8+22}\right)$.

$w(x) = \left(\tan\left((x^8+22)^{1/2}\right)\right)^5$

$w'(x) = 5\left(\tan\left((x^8+22)^{1/2}\right)\right)^4\cdot\frac{d}{dx}\left(\tan\left((x^8+22)^{1/2}\right)\right)$

$w'(x) = 5\left(\tan\left((x^8+22)^{1/2}\right)\right)^4\cdot\sec^2\left((x^8+22)^{1/2}\right)\cdot\frac{d}{dx}\left((x^8+22)^{1/2}\right)$

$w'(x) = 5\left(\tan\left((x^8+22)^{1/2}\right)\right)^4\cdot\sec^2\left((x^8+22)^{1/2}\right)\cdot\frac{1}{2}(x^8+22)^{-1/2}\cdot\frac{d}{dx}(x^8+22)$

$w'(x) = 5\left(\tan\left((x^8+22)^{1/2}\right)\right)^4\cdot\sec^2\left((x^8+22)^{1/2}\right)\cdot\frac{1}{2}(x^8+22)^{-1/2}\cdot 8x^7$

$$w'(x) = \frac{20x^7\tan^4\left(\sqrt{x^8+22}\right)\sec^2\left(\sqrt{x^8+22}\right)}{\sqrt{x^8+22}}$$

3. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

Method 1

$$y = \left(x^3 + 5\right)^{1/x^2}$$

$$\ln(y) = \ln\left((x^3+5)^{1/x^2}\right)$$

$$\ln(y) = \frac{1}{x^2}\ln(x^3+5)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(\frac{\ln(x^3+5)}{x^2}\right)$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{\frac{1}{x^3+5}\cdot 3x^2\cdot x^2 - \ln(x^3+5)\cdot 2x}{(x^2)^2}$$

$$\frac{dy}{dx} = y\left(\frac{\frac{3x^4}{x^3+5} - 2x\ln(x^3+5)}{x^4}\right)$$

$$\boxed{\frac{dy}{dx} = (x^3+5)^{1/x^2}\left(\frac{\frac{3x^4}{x^3+5} - 2x\ln(x^3+5)}{x^4}\right)}$$

Method 2

$$y = (x^3+5)^{1/x^2}$$

$$y = e^{\ln\left((x^3+5)^{1/x^2}\right)}$$

$$y = e^{\frac{1}{x^2}\ln(x^3+5)}$$

$$\frac{dy}{dx} = e^{\frac{1}{x^2}\ln(x^3+5)}\cdot\frac{d}{dx}\left(\frac{\ln(x^3+5)}{x^2}\right)$$

$$= e^{\ln\left((x^3+5)^{1/x^2}\right)}\cdot\left(\frac{\frac{1}{x^3+5}\cdot 3x^2\cdot x^2 - \ln(x^3+5)\cdot 2x}{(x^2)^2}\right)$$

$$= (x^3+5)^{1/x^2}\left(\frac{\frac{3x^4}{x^3+5} - 2x\ln(x^3+5)}{x^4}\right)$$

4. (3 points) Compute $\frac{dy}{dx}$ given that $x^2 e^{3y} = \ln\left(x^3 y^2\right)$.

$$x^2 e^{3y} = \ln(x^3) + \ln(y^2)$$

$$x^2 e^{3y} = 3\ln(x) + 2\ln(y)$$

$$\frac{d}{dx}\left(x^2 e^{3y}\right) = \frac{d}{dx}\left(3\ln(x) + 2\ln(y)\right)$$

your answer may be correct but look different if you did not simplify first

$$2xe^{3y} + x^2 e^{3y} \cdot 3\frac{dy}{dx} = 3\cdot\frac{1}{x} + 2\cdot\frac{1}{y}\frac{dy}{dx}$$

$$3x^2 e^{3y}\frac{dy}{dx} - \frac{2}{y}\frac{dy}{dx} = \frac{3}{x} - 2xe^{3y}$$

$$\frac{dy}{dx}\left(3x^2 e^{3y} - \frac{2}{y}\right) = \frac{3}{x} - 2xe^{3y}$$

$$\frac{dy}{dx} = \frac{\frac{3}{x} - 2xe^{3y}}{3x^2 e^{3y} - \frac{2}{y}}$$