Math 220 (section AD?) Quiz 4 Spring 2016

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) Determine the equation of the line which is tangent to $f(x) = 7 - 5e^x$ and perpendicular to the line $y = \frac{1}{10}x + 3$.

since $y=\frac{1}{10}x+3$ is a line with slope $\frac{1}{10}$, a line perpendicular to it has slope $\frac{-1}{1/10} = -10$.

$f'(x) = -5e^x$ gives a formula for the slope of each line tangent to $f(x)$.

we set $f'(x) = -10$

$-5e^x = -10 \Rightarrow e^x = 2 \Rightarrow x = \ln(2)$

point: $(\ln(2), f(\ln(2))) = (\ln(2), 7-5e^{\ln(2)}) = (\ln(2), -3)$

slope: -10

tangent line: $y-(-3) = -10(x-\ln(2))$

$y = -10x + 10\ln(2) - 3$

2. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $w = \left(\frac{x^2\sqrt{x}}{\sqrt[5]{x}}\right)^{20} + 4e^{\sin^3(\pi/12)}$ (simplify your answer)

$$w = \frac{(x^2\sqrt{x})^{20}}{(\sqrt[5]{x})^{20}} + 4e^{\sin^3(\pi/12)}$$

$$w = \frac{(x^2)^{20}(\sqrt{x})^{20}}{(\sqrt[5]{x})^{20}} + 4e^{\sin^3(\pi/12)}$$

$$w = \frac{x^{40}x^{10}}{x^4} + 4e^{\sin^3(\pi/12)}$$

$$w = x^{46} + 4e^{\sin^3(\pi/12)}$$

$$\frac{dw}{dx} = 46x^{45}$$

note: $4e^{\sin^3(\pi/12)}$ is a constant

(b) $P = s^8 \cot(s)$

$$\frac{dP}{ds} = \frac{d}{ds}(s^8)\cdot\cot(s) + s^8\cdot\frac{d}{ds}(\cot(s))$$

$$= 8s^7\cdot\cot(s) + s^8\cdot(-\csc^2(s))$$

$$= 8s^7\cot(s) - s^8\csc^2(s)$$

(c) $\theta = \frac{\sqrt[3]{t}+10}{6t^2+\sec(t)}$ $= \frac{t^{1/3}+10}{6t^2+\sec(t)}$

$$\frac{d\theta}{dt} = \frac{\frac{d}{dt}(t^{1/3}+10)\cdot(6t^2+\sec(t)) - (t^{1/3}+10)\cdot\frac{d}{dt}(6t^2+\sec(t))}{(6t^2+\sec(t))^2}$$

$$= \frac{\frac{1}{3}t^{-2/3}(6t^2+\sec(t)) - (t^{1/3}+10)(12t+\sec(t)\tan(t))}{(6t^2+\sec(t))^2}$$