Math 220 (section AD?) Quiz 3 Spring 2016

Name Solutions

- 20 minutes
- No calculators
- Show sufficient work
- Do not use derivatives

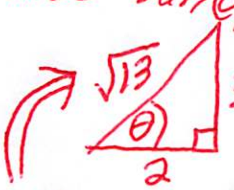
1. (2 points) Evaluate $\csc(\arctan(3/2))$.

Let $\theta = \arctan\left(\frac{3}{2}\right)$

Then $\tan(\theta) = 3/2$ $\left(\frac{\text{opp}}{\text{adj}}\right)$

and θ is in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Actually, θ is in $\left(0, \frac{\pi}{2}\right)$ since $\tan(\theta) > 0$



Pythagorean Theorem $\left(2^2 + 3^2 = (\text{hyp})^2,\ \text{hyp} = \sqrt{13}\right)$

$\csc\left(\arctan\left(\frac{3}{2}\right)\right) = \csc(\theta)$ $\left(\frac{\text{hyp}}{\text{opp}}\right)$

$= \frac{\sqrt{13}}{3}$

Alternate Solution:

$\csc(\arctan(3/2)) = \sqrt{\csc^2(\arctan(3/2))}$

$= \sqrt{\cot^2(\arctan(3/2)) + 1}$

$= \sqrt{\frac{1}{\tan^2(\arctan(3/2))} + 1}$

$= \sqrt{\frac{1}{(3/2)^2} + 1} = \frac{\sqrt{13}}{3}$

2. (2 points) For what value of the constant C is the function f continuous at $x = 0$?

$$f(x) = \begin{cases} \dfrac{\sin(2x)}{x\cos(x)} & \text{if} \quad x < 0 \\ 9e^x + C & \text{if} \quad x \geq 0 \end{cases}$$

$\boxed{\text{We need } \lim_{x \to 0} f(x) = f(0)}$

$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \frac{\sin(2x)}{x\cos(x)} = \lim_{x \to 0^-} \frac{2\sin(x)\cos(x)}{x\cos(x)} = 2 \cdot \lim_{x \to 0} \frac{\sin(x)}{x} = 2 \cdot 1 = 2$

$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (9e^x + C) = 9e^0 + C = 9 + C$

$f(0) = 9e^0 + C = 9 + C$

all must be equal

$9 + C = 2 \Rightarrow C = -7$

3. (2 points each) Evaluate the following limits. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\lim_{x\to 3^+} \frac{x^2-8x+15}{x^2-6x+9}$ $\nearrow 0$ $\to 0$ $= \lim_{x\to 3^+} \frac{(x-3)(x-5)}{(x-3)^2}$

$= \lim_{x\to 3^+} \frac{x-5}{x-3}$ $\nearrow -2$ $\to 0^+$

$= -\infty$

(b) $\lim_{x\to 0} \frac{2x}{\sqrt{25+x}-5}$ $\nearrow 0$ $\to 0$ $= \lim_{x\to 0} \frac{2x}{\sqrt{25+x}-5}\cdot\frac{\sqrt{25+x}+5}{\sqrt{25+x}+5}$

$= \lim_{x\to 0} \frac{2x(\sqrt{25+x}+5)}{(\sqrt{25+x})^2-5^2}$

$= \lim_{x\to 0} \frac{2x(\sqrt{25+x}+5)}{25+x-25}$

$= \lim_{x\to 0} \frac{2x(\sqrt{25+x}+5)}{x}$

$= \lim_{x\to 0} 2(\sqrt{25+x}+5)$

$= 2(\sqrt{25+0}+5)$

$= 20$

4. (2 points) Determine an equation for each horizontal asymptote on the graph of the function.

$$f(x) = \frac{6e^x}{3e^x + 4}$$

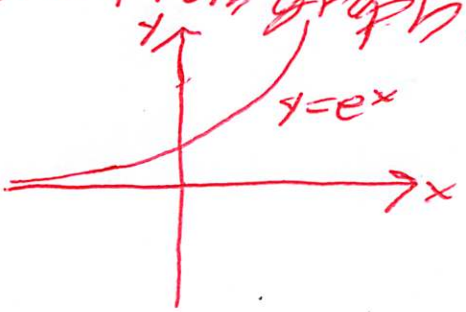
$$\lim_{x\to\infty} \frac{6e^x \to \infty}{3e^x+4 \to \infty} = \lim_{x\to\infty} \frac{6e^x}{3e^x+4} \cdot \frac{1/e^x}{1/e^x}$$

$$= \lim_{x\to\infty} \frac{6}{3+4/e^x}$$

$$= \frac{6}{3+0} = 2$$

$$\lim_{x\to-\infty} \frac{6e^x \to 6\cdot 0}{3e^x+4 \to 3\cdot 0+4} = \frac{0}{4} = 0$$

note from graph: $\lim_{x\to-\infty} e^x = 0$



f(x) has two horizontal asymptotes.
$y=2$ and $y=0$