

Name

Solutions

• You have 15 minutes

• No calculators

• Show sufficient work

1. (4 points) The following curves intersect. Find the x -value for each point of intersection.

$$y = x^2 \ln(9 - 2x) + 20$$

$$y = 100 \ln(9 - 2x) + 20$$

$$x^2 \ln(9 - 2x) + 20 = 100 \ln(9 - 2x) + 20$$

$$x^2 \ln(9 - 2x) = 100 \ln(9 - 2x)$$

$$x^2 \ln(9 - 2x) - 100 \ln(9 - 2x) = 0$$

$$(x^2 - 100) \ln(9 - 2x) = 0$$

$$(x - 10)(x + 10) \ln(9 - 2x) = 0$$

$$x - 10 = 0 \Rightarrow x = 10 \quad \left(\begin{array}{l} \text{not in domain of} \\ \text{either function} \end{array} \right)$$

$$x + 10 = 0 \Rightarrow x = -10 \quad (\text{in domain})$$

$$\ln(9 - 2x) = 0 \Rightarrow e^{\ln(9 - 2x)} = e^0$$

$$\Rightarrow 9 - 2x = 1$$

$$\Rightarrow x = 4$$

$$\Rightarrow x = 4 \quad (\text{in domain})$$

There are two points of intersection.
The x -values are -10 and 4

2. (3 points) Some of the values for an exponential function $y = f(x)$ are shown below. Determine a formula for this function.

x	$f(x)$
6	2
9	10
12	50
15	250
18	1250

$$y = C \cdot a^x$$

~~at~~ at $(x, y) = (6, 2)$ we have

$$2 = C \cdot a^6 \Rightarrow C = \frac{2}{a^6}$$

at $(x, y) = (9, 10)$ we have

$$10 = C \cdot a^9$$

$$10 = \left(\frac{2}{a^6}\right) \cdot a^9 \Rightarrow 10 = 2a^3$$

$$\Rightarrow 5 = a^3$$

$$\Rightarrow a = \sqrt[3]{5} = 5^{1/3}$$

$$C = \frac{2}{(5^{1/3})^6} = \frac{2}{25}$$

$$y = \frac{2}{25} \cdot (5^{1/3})^x$$

$$f(x) = \frac{2}{25} \cdot 5^{x/3}$$

3. (3 points) If $p(x) = \frac{5}{\sqrt[3]{4e^x+9}}$ then determine a formula for $p^{-1}(x)$.

$$\text{Let } y = p(x)$$

$$y = \frac{5}{\sqrt[3]{4e^x+9}}$$

$$x = \frac{5}{\sqrt[3]{4e^y+9}}$$

(switch x & y)

$$x(\sqrt[3]{4e^y+9}) = 5$$

(solve for y)

$$\sqrt[3]{4e^y+9} = \frac{5}{x}$$

$$4e^y+9 = \left(\frac{5}{x}\right)^3$$

$$4e^y = \left(\frac{5}{x}\right)^3 - 9$$

$$e^y = \frac{\left(\frac{5}{x}\right)^3 - 9}{4}$$

$$\ln(e^y) = \ln\left(\frac{\left(\frac{5}{x}\right)^3 - 9}{4}\right)$$

$$y = \ln\left(\frac{\left(\frac{5}{x}\right)^3 - 9}{4}\right)$$

$$p^{-1}(x) = \ln\left(\frac{\left(\frac{5}{x}\right)^3 - 9}{4}\right)$$