Math 220 **Quiz 12 (take-home)** **Spring 2016**

Name solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Hannah Burson
- ▷ **AD2**, TR 1:00-2:50, Cassie Christenson
- ▷ **ADA**, TR 8:00-8:50, Iftikhar Ahmed
- ▷ **ADB**, TR 9:00-9:50, Iftikhar Ahmed
- ▷ **ADC**, TR 10:00-10:50, Elizabeth 'Liz' Tatum
- ▷ **ADD**, TR 11:00-11:50, Elizabeth 'Liz' Tatum
- ▷ **ADE**, TR 12:00-12:50, Emily Heath
- ▷ **ADF**, TR 1:00-1:50, Emily Heath
- ▷ **ADG**, TR 2:00-2:50, Dara Zirlin
- ▷ **ADH**, TR 3:00-3:50, Dara Zirlin
- ▷ **ADJ**, TR 9:00-9:50, Xujun 'Henry' Liu
- ▷ **ADK**, TR 10:00-10:50, Xujun 'Henry' Liu
- ▷ **ADL**, TR 11:00-11:50, Jooyeon 'Jane' Chung
- ▷ **ADM**, TR 12:00-12:50, Jooyeon 'Jane' Chung
- ▷ **ADN**, TR 1:00-1:50, Xiaolong 'Hans' Han
- ▷ **ADO**, TR 2:00-2:50, Martino Fassina
- ▷ **ADP**, TR 3:00-3:50, Martino Fassina
- ▷ **ADQ**, TR 4:00-4:50, Xiaolong 'Hans' Han

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- You may use a calculator only for basic arithmetic.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **The quiz is due at the beginning of your official lecture period on Friday, April 22.**
- **Note to TAs and Tutors – you should not help students with these specific problems until I post solutions Friday evening.**

1. (3 points) A calculator gives an estimate of 0.8187307531 for the value of $\dfrac{e^2}{\sqrt[5]{e^{11}}}$.

Using the techniques of linear approximation found in section 3.10, show that you are able to obtain a very similar estimate of 0.8 without the use of any technology.

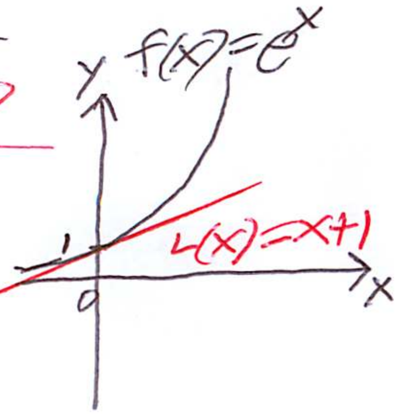
$$\frac{e^2}{\sqrt[5]{e^{11}}} = \frac{e^2}{e^{11/5}} = e^{2-11/5} = e^{-1/5}$$

Tangent line to $f(x)=e^x$ at $x=0$

Point: $(0, f(0)) = (0, e^0) = (0,1)$

slope: $f'(0) = e^0 = 1$ (since $f'(x)=e^x$)

tangent line: $L(x) = x+1$



$f(x) \approx L(x)$ for x near the point of tangency.

$e^x \approx x+1$ for x near 0

$e^{-1/5} \approx -\frac{1}{5}+1$ (from graph, we see this is an underestimate)

$e^{-1/5} \approx 0.8$

2. (3 points) Let $g(x) = \int_{-125}^{x^3} f(t)\,dt$.

Use the techniques of linear approximation found in section 3.10 to approximate $g(5.6)$ given the following information about f.

- f is continuous on the interval $(-\infty, \infty)$
- f is an odd function
- $f(125) = \frac{1}{150}$

$$g(5) = \int_{-125}^{5^3} f(t)\,dt = \int_{-125}^{125} f(t)\,dt = 0$$

since f is odd and cont.

From the Fundamental Theorem of Calculus (Part 1),

$$g'(x) = f(x^3) \cdot 3x^2$$

Thus, $g'(5) = f(125) \cdot 75 = \frac{1}{150} \cdot 75 = \frac{1}{2}$

Tangent line at $x = 5$

Point: $(5, g(5)) = (5, 0)$

Slope: $g'(5) = \frac{1}{2}$

$$y - 0 = \frac{1}{2}(x-5)$$

$$y = \frac{1}{2}(x-5)$$

$g(x) \approx \frac{1}{2}(x-5)$ for x near 5

$g(5.6) \approx \frac{1}{2}(5.6-5) = 0.3$

3. (4 points) The function $g(x) = x^4 + 3x^2 - 5x$ has precisely one critical number. Determine the value of this critical number using Newton's Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

$g'(x) = 4x^3 + 6x - 5$

This derivative exists for all x so the only critical numbers occur when $g'(x) = 0$. We need to solve $4x^3 + 6x - 5 = 0$

Let $f(x) = 4x^3 + 6x - 5$ (thus $f'(x) = 12x^2 + 6$) and apply Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = x_n - \frac{4x_n^3 + 6x_n - 5}{12x_n^2 + 6}$$

$x_1 = 1$ given as first estimate

$$x_2 = 1 - \frac{4(1)^3 + 6(1) - 5}{12(1)^2 + 6} = \frac{13}{18} \approx 0.7222222222$$

$$x_3 \approx 0.7222222222 - \frac{4(0.7222222222)^3 + 6(0.7222222222) - 5}{12(0.7222222222)^2 + 6}$$

$x_3 \approx 0.6536869195$

$$x_4 \approx 0.6536869195 - \frac{4(0.6536869195)^3 + 6(0.6536869195) - 5}{12(0.6536869195)^2 + 6}$$

$x_4 \approx 0.6501443634$