Math 220 (section AD?) Quiz 11 Spring 2016

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{24x}{\sqrt{4x^2+9}}$ on the interval $[0, 2]$. Simplify your answer.

$$f_{ave} = \frac{1}{2-0}\int_0^2 \frac{24x}{\sqrt{4x^2+9}}\,dx$$

$$= \frac{1}{2}\int_9^{25} \frac{3\,du}{\sqrt{u}}$$

$$= \frac{3}{2}\int_9^{25} u^{-1/2}\,du$$

$$= \frac{3}{2}\cdot\left[\frac{1}{1/2}u^{1/2}\right]_9^{25}$$

$$= \frac{3}{2}\left[2\sqrt{u}\right]_9^{25}$$

$$= \frac{3}{2}\left(2\sqrt{25} - 2\sqrt{9}\right)$$

$$= \frac{3}{2}(10 - 6)$$

$$= 6$$

$u = 4x^2+9$

$du = 8x\,dx$

$3\,du = 24x\,dx$

at $x=0$,
$u = 4\cdot 0^2+9 = 9$

at $x=2$,
$u = 4\cdot 2^2+9 = 25$

2. Let **R** be the finite region bounded by the graphs of $x^2 + 9y = 0$ and $x + 3y = 0$. Set up, but do not evaluate, definite integrals which represent the volumes of the following solids.

   (a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the x-axis are squares.

$$x^2+9y=0 \Rightarrow y=-\frac{1}{9}x^2$$

$$x+3y=0 \Rightarrow y=-\frac{1}{3}x$$

intersection

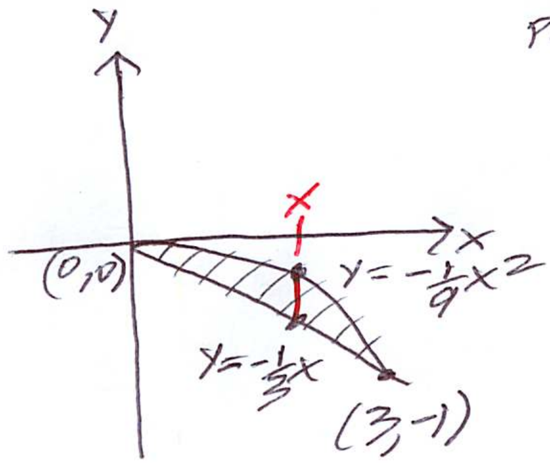
$$-\frac{1}{9}x^2=-\frac{1}{3}x \Rightarrow x^2=3x$$

$$x^2-3x=0$$

$$x(x-3)=0$$

$$x=0, \quad x=3$$

Point $(0,0)$ Point $(3,-1)$



cross-section at x is a square

with area (side length)^2

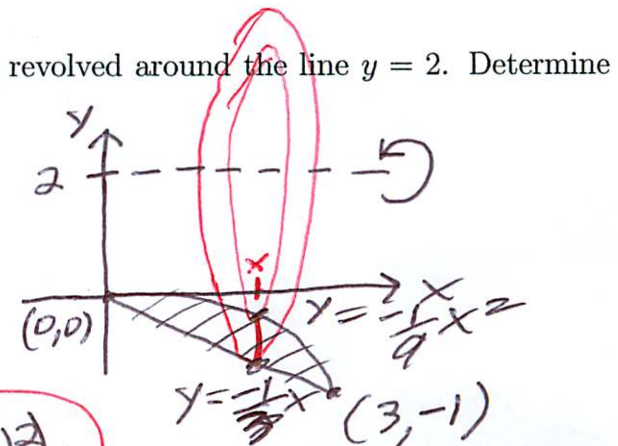
$$V=\int_{x_{min}}^{x_{max}}\begin{pmatrix}\text{cross-sectional}\\ \text{area}\end{pmatrix}dx$$

$$=\int_0^3 (\text{side length})^2\,dx$$

$$=\int_0^3 \left(\underbrace{-\frac{1}{9}x^2}_{y_{top}}-\underbrace{\left(-\frac{1}{3}x\right)}_{y_{bottom}}\right)^2 dx$$

(b) The volume of the solid formed when **R** is revolved around the line $y = 2$. Determine this volume in the following two ways.

i. (2 points) Integrate with respect to x.



$$V = \int_{x_{min}}^{x_{max}} \begin{pmatrix}\text{cross-sectional}\\ \text{area}\end{pmatrix} dx$$

$$= \int_0^3 \left(\pi\big(\underbrace{2-(-\tfrac{1}{3}x)}_{\text{big radius}}\big)^2 - \pi\big(\underbrace{2-(-\tfrac{1}{9}x^2)}_{\text{small radius}}\big)^2\right) dx$$

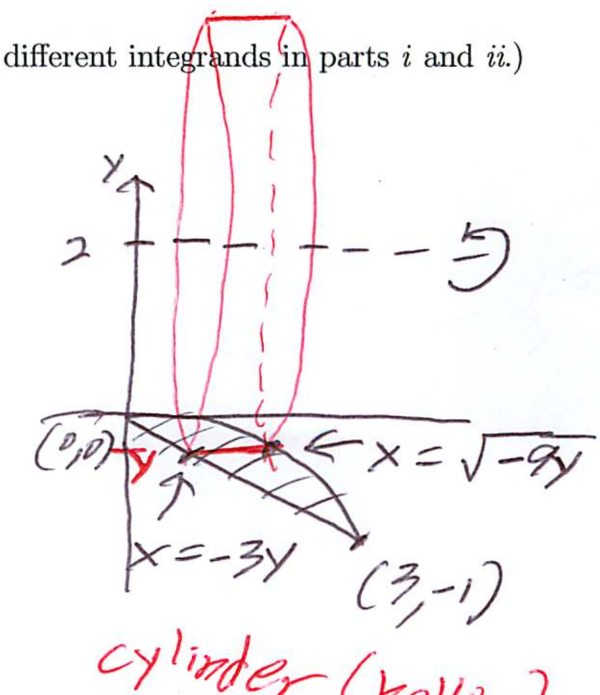
cross-section at x

has area $\pi \cdot r_{out}^2 - \pi \cdot r_{in}^2$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)

$$V = \int_{y_{min}}^{y_{max}} \begin{pmatrix}\text{surface}\\ \text{area}\end{pmatrix} dy$$

$$= \int_{-1}^{0} 2\pi\underbrace{(2-y)}_{\text{radius}}\underbrace{\left(\sqrt{-9y} - (-3y)\right)}_{\text{height}} dy$$



cylinder (hollow) has surface area $2\pi r h$

note:
$x^2+9y=0 \Rightarrow x=\pm\sqrt{-9y}$
$x+3y=0 \Rightarrow x=-3y$