Math 220 (section AD?) Quiz 10 Spring 2016

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

Suppose f is continuous on $[a,b]$ and differentiable on (a,b).

Then $f'(c) = \frac{f(b)-f(a)}{b-a}$

for some c in (a,b).

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_4^{25} \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx = \int_4^{25} e^{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}\,dx$$

$$= \int_2^5 e^u \cdot 2\,du$$

$$= 2\int_2^5 e^u\,du$$

$$= 2\left[e^u\right]_2^5$$

$$= 2\left[e^5 - e^2\right]$$

$$= 2(e^5 - e^2)$$

u-substitution

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx$

$2du = \frac{1}{\sqrt{x}}dx$

at $x=4$, $u = \sqrt{4} = 2$

at $x = 25$, $u = \sqrt{25} = 5$

3. (2 points each) Evaluate the following integrals.

(a) $\int \frac{50x^9 \cos(x^{10})}{\sin(x^{10})}\,dx = \int \frac{5\cos(x^{10})}{\sin(x^{10})} \cdot 10x^9\,dx$ $\left(\begin{array}{l} u = x^{10} \\ du = 10x^9\,dx \end{array}\right)$

$= \int \frac{5\cos(u)}{\sin(u)}\,du$

$= \int \frac{5}{\sin(u)} \cdot \cos(u)\,du$ $\left(\begin{array}{l} w = \sin(u) \\ dw = \cos(u)\,du \end{array}\right)$

$= \int \frac{5}{w}\,dw$

$= 5\int \frac{1}{w}\,dw$

$= 5\ln|w| + C$

$= 5\ln|\sin(u)| + C$

$= 5\ln|\sin(x^{10})| + C$

note: Letting $u = \sin(x^{10})$ allows us to solve with only one substitution.

(b) $\int 40x\sqrt{2x+3}\,dx = \int 20x\sqrt{2x+3} \cdot 2\,dx$ $\left(\begin{array}{l} u = 2x+3 \\ du = 2\,dx \\ \text{note that} \\ u = 2x+3 \Rightarrow \\ x = \frac{u-3}{2} \end{array}\right)$

$= \int 20\left(\frac{u-3}{2}\right)\sqrt{u}\,du$

$= \int 10(u-3)\,u^{1/2}\,du$

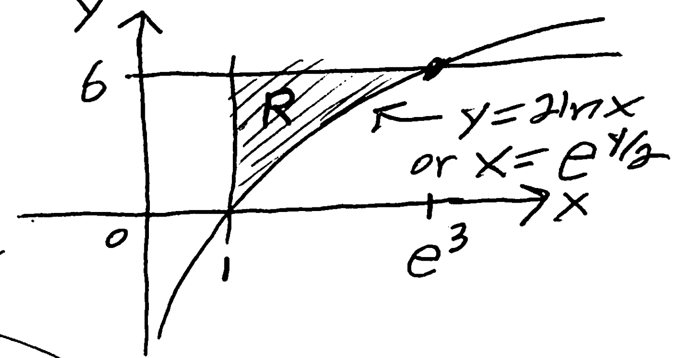
$= \int \left(10u^{3/2} - 30u^{1/2}\right)du$

$= 10 \cdot \frac{1}{5/2}u^{5/2} - 30 \cdot \frac{1}{3/2}u^{3/2} + C$

$= 4u^{5/2} - 20u^{3/2} + C$

$= 4(2x+3)^{5/2} - 20(2x+3)^{3/2} + C$

4. (2 points) Let **R** be the finite region bounded by $y = 2\ln x$, $y = 6$ and $x = 1$. In the following manner, set up but do not evaluate definite integrals which represent the area of the region **R**.



(a) Integrate with respect to x.

$$\text{area} = \int_{x_{min}}^{x_{max}} (y_{top} - y_{bottom})\,dx$$

$$= \int_1^{e^3} (6 - 2\ln x)\,dx$$

note:
$y = 2\ln x$ and $y = 6$
intersect when
$2\ln x = 6 \Rightarrow$
$\ln x = 3 \Rightarrow$
$x = e^3$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$\text{area} = \int_{y_{min}}^{y_{max}} (x_{right} - x_{left})\,dy$$

$$= \int_0^6 (e^{y/2} - 1)\,dy$$

note:
solving
$y = 2\ln x$
for x gives
$\frac{y}{2} = \ln x \Rightarrow$
$x = e^{y/2}$