Math 220 (section AD?) Quiz 1 Spring 2016

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Given an acute angle θ for which $\cos(\theta) = 2/3$, evaluate the following quantities.

(a) $\sin(\theta)$

method 1

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\left(\frac{2}{3}\right)^2 + \sin^2(\theta) = 1$

$\sin^2(\theta) = 1 - \frac{4}{9} = \frac{5}{9}$

$\sin(\theta) = \pm\sqrt{5/9}$

θ is acute $\Rightarrow \sin(\theta) = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$

method 2

$h = \sqrt{5}$ $\quad \cos(\theta) = \frac{2}{3}$ $\left(\frac{adj}{hyp}\right)$

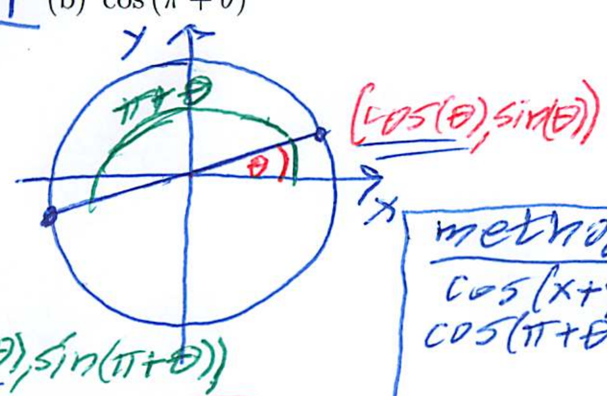
Pyth. Thm. $\Rightarrow$

$2^2 + h^2 = 3^2 \Rightarrow h = \sqrt{5}$

$\sin(\theta) = \frac{\sqrt{5}}{3}$ $\left(\frac{opp}{hyp}\right)$

(b) $\cos(\pi + \theta)$

method 1
unit circle



$\cos(\pi + \theta) = -\cos(\theta)$

$= -\frac{2}{3}$

method 2

$\cos(x+y) = \cos(x)\cos(y) - \sin(x)\sin(y)$

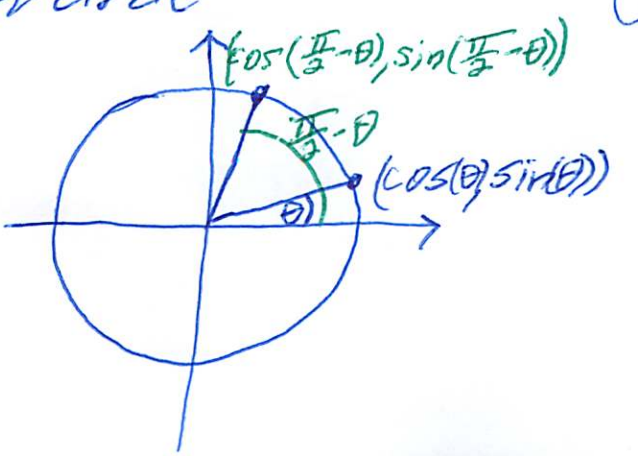
$\cos(\pi + \theta) = \cos(\pi)\cos(\theta) - \sin(\pi)\sin(\theta)$

$= -1 \cdot \cos(\theta) - 0 \cdot \sin(\theta) = -\cos(\theta) = -\frac{2}{3}$

(c) $\sin\left(\frac{\pi}{2} - \theta\right)$

method 1
unit circle

$\sin\left(\frac{\pi}{2} - \theta\right) = \cos(\theta) = \frac{2}{3}$



method 2

$\sin(x-y) = \sin(x)\cos(y) - \cos(x)\sin(y)$

$\sin\left(\frac{\pi}{2} - \theta\right) = \sin\left(\frac{\pi}{2}\right)\cos(\theta) - \cos\left(\frac{\pi}{2}\right)\sin(\theta)$

$= 1 \cdot \cos(\theta) - 0 \cdot \sin(\theta)$

$= \cos(\theta)$

$= \frac{2}{3}$

2. (4 points) Determine the domain of the given function.

$$f(x) = \frac{\sqrt{x^2+4}}{\sqrt{25-4x}-\sqrt{x+9}}$$

① From $\sqrt{x^2+4}$, $x^2+4 \geq 0$ (always true, no restriction on domain)

② From $\sqrt{25-4x}$, $25-4x \geq 0 \Rightarrow -4x \geq -25$

$\Rightarrow x \leq \frac{25}{4}$

③ From $\sqrt{x+9}$, $x+9 \geq 0 \Rightarrow x \geq -9$

④ denominator $=0$ when $\sqrt{25-4x} - \sqrt{x+9} = 0$

$\Rightarrow \sqrt{25-4x} = \sqrt{x+9}$

$\Rightarrow 25-4x = x+9$

$\Rightarrow 16 = 5x$

$\Rightarrow x = 16/5$

Thus $x \neq 16/5$

domain of f: $[-9, \frac{16}{5}) \cup (\frac{16}{5}, \frac{25}{4}]$

3. (3 points) Determine whether the following function is even, odd or neither. Give a very clear justification for your answer.

$$g(t) = \cos\left(t^3\right) - \sin\left(t^4\right)$$

$$
\begin{aligned}
g(-t) &= \cos\left((-t)^3\right) - \sin\left((-t)^4\right) \\
&= \cos\left(-t^3\right) - \sin\left(t^4\right) \\
&= \cos\left(t^3\right) - \sin\left(t^4\right) \quad \text{(since cosine is even)} \\
&= g(t)
\end{aligned}
$$

g is an even function