• Sit in your assigned seat (circled below).
• Circle your TA discussion section.
• Do not open this test booklet until I say START.
• Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
• Remove hats and sunglasses.
• You must show sufficient work to justify each answer.
• While the test is in progress, we will not answer questions concerning the test material.
• Do not leave early unless you are at the end of a row.
• Quit working and close this test booklet when I say STOP.
• Quickly turn in your test to me or a TA and show your Student ID.

AD1, TR 9:00-10:50, Andrew McConvey
AD2, TR 1:00-2:50, Derrek Yager
ADA, TR 8:00-8:50, Mi Young Jang
ADB, TR 9:00-9:50, Stephen Berning
ADC, TR 10:00-10:50, Sarah Yeakel
ADD, TR 11:00-11:50, Michael Livesay
ADE, TR 12:00-12:50, George Shakan
ADF, TR 1:00-1:50, Albert Tamazyan
ADG, TR 2:00-2:50, Alonza Terry
ADH, TR 3:00-3:50, Alonza Terry

ADJ, TR 9:00-9:50, Mi Young Jang
ADK, TR 10:00-10:50, Stephen Berning
ADL, TR 11:00-11:50, Adam Wagner
ADM, TR 12:00-12:50, Adam Wagner
ADN, TR 1:00-1:50, Mychael Sanchez
ADO, TR 2:00-2:50, Mychael Sanchez
ADP, TR 3:00-3:50, Albert Tamazyan
ADQ, TR 4:00-4:50, George Shakan
ADR, TR 9:00-9:50, Michael Livesay
1. (12 points) Let \( f(x) = x^3 - 42x \).

Use the definition of a derivative as a limit to prove that \( f'(x) = 3x^2 - 42 \).

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

\[
\begin{align*}
\frac{d}{dx}(x^3 - 42x) &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\
&= \lim_{h \to 0} \frac{(x+h)^3 - 42(x+h) - (x^3 - 42x)}{h} \\
&= \lim_{h \to 0} \frac{x^3 + 3x^2h + 3xh^2 + h^3 - 42x - 42h - x^3 + 42x}{h} \\
&= \lim_{h \to 0} \frac{3x^2h + 3xh^2 + h^3 - 42h}{h} \\
&= \lim_{h \to 0} \frac{h(3x^2 + 3xh + h^2 - 42)}{h} \\
&= \lim_{h \to 0} (3x^2 + 3xh + h^2 - 42) \\
&= 3x^2 - 42
\end{align*}
\]
2. (12 points) The function \( f(x) = 20e^{5x} + 15x - 12 \) has derivative \( f'(x) = 100e^{5x} + 15 \). Determine a formula for the line which is tangent to the graph of \( f(x) \) at its \( y \)-intercept.

Point: \((0, f(0)) = (0, 8)\)

slope: \( f'(0) = 115 \)

tangent line: \( y = 115x + 8 \)

3. (12 points) Let \( R(t) \) be the number of rabbits living on Lady Tottington’s estate \( t \) months after they were initially discovered. This rabbit population grows exponentially. Given that \( R(2) = 10 \) and \( R(5) = 90 \), determine a formula for \( R(t) \).

\[
R(t) = C \cdot a^t
\]

\( R(2) = 10 \Rightarrow 10 = C \cdot a^2 \Rightarrow C = \frac{10}{a^2} \)

\( R(5) = 90 \Rightarrow 90 = C \cdot a^5 \)

Thus \( 90 = \left( \frac{10}{a^2} \right) a^5 \)

\( 9 = a^3 \)

\( a = 9^{\frac{1}{3}} = 3^{\frac{2}{3}} \)

\( C = \frac{10}{\left(3^{\frac{2}{3}}\right)^2} = \frac{10}{3^{\frac{4}{3}}} \)

\[
R(t) = \left( \frac{10}{3^{\frac{4}{3}}} \right) \cdot \left(3^{\frac{2}{3}}\right)^t
\]
4. (12 points) Determine a formula for \( g^{-1}(x) \) given that \( g(x) = \frac{8x^9 - 3}{5x^9 + 4} \)

\[
y = \frac{8x^9 - 3}{5x^9 + 4}
\]

\[
x = \frac{8y^9 - 3}{5y^9 + 4} \quad (\text{switch } x \& y)
\]

\[
x(5y^9 + 4) = 8y^9 - 3 \quad (\text{solve for } y)
\]

\[
5xy^9 + 4x = 8y^9 - 3
\]

\[
5xy^9 - 8y^9 = -3 - 4x
\]

\[
y^9(5x - 8) = -3 - 4x
\]

\[
y^9 = \frac{-3 - 4x}{5x - 8}
\]

\[
y = \left( \frac{-3 - 4x}{5x - 8} \right)^{\frac{1}{9}}
\]

\[
g^{-1}(x) = \left( \frac{-3 - 4x}{5x - 8} \right)^{\frac{1}{9}}
\]

5. (12 points) Solve the following equation for \( x \) and simplify your answer.

\[
\ln(2) + 9 \ln(-x) = \ln(-128x^7)
\]

\[
\ln(2) + \ln((-x)^9) = \ln(-128x^7)
\]

\[
\ln(-2x^9) = \ln(-128x^7)
\]

\[
-2x^9 = -128x^7
\]

\[
x^9 = 64x^7
\]

\[
x^9 - 64x^7 = 0
\]

\[
x^7(x - 8)(x + 8) = 0
\]

\[
x = 0 \text{ or } x = 8 \text{ or } x = -8
\]

Only \( x = -8 \) is in the domain of both sides in the original equation.
6. (10 points) Suppose that $w(x)$ is odd, one-to-one, and its graph goes through the point $(4, -1/3)$.

(a) Determine another point which must be on the graph of $w(x)$.

$$w(-x) = -w(x)$$

Since $w$ is odd. Thus $(4, \frac{1}{3})$ is a point on $w(x)$.

(b) Determine a point which must be on the graph of $w^{-1}(x)$.

$(4, -\frac{1}{3})$ on $w^{-1}(x) \Rightarrow (\frac{1}{3}, 4)$ on $w^{-1}(x)$

(Also from (a))

$(4, \frac{1}{3})$ on $w(x) \Rightarrow (\frac{1}{3}, -4)$ on $w^{-1}(x)$

7. (5 points) Given that $\cos(\pi/5) = \frac{1 + \sqrt{5}}{4}$, evaluate $\cos(4\pi/5)$.

$$\cos\left(\frac{4\pi}{5}\right) = \cos\left(\pi - \frac{\pi}{5}\right) = -\cos\left(\frac{\pi}{5}\right) = -\frac{1 + \sqrt{5}}{4}.$$  

We used that $\cos(\pi - \theta) = -\cos(\theta)$.

8. (5 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of ‘does not exist’ is not sufficient. For infinite limits you must state if it is $\infty$ or $-\infty$.

(a) $\lim_{x\to\infty} \frac{(2x + 1)^5}{4 + 3x^5}$

$$= \lim_{x\to\infty} \frac{(2x+1)^5}{x^5} \cdot \frac{1}{(4+3x^5)} \cdot \frac{1}{x^5}$$

$$= \lim_{x\to\infty} \frac{(2x+1)^5}{x^5} \cdot \frac{1}{4x^3 + 3}$$

$$= \lim_{x\to\infty} \frac{(2+\frac{1}{x})^5}{4 + 3} \to 2^5$$

$$= \frac{2^5}{3} = \frac{32}{3}$$
(b) \[ \lim_{x \to \infty} \frac{\cos(2x)}{x^{10}} \]

\[-1 \leq \cos(2x) \leq 1\]

\[-\frac{1}{x^{10}} \leq \frac{\cos(2x)}{x^{10}} \leq \frac{1}{x^{10}}\]

\[\lim_{x \to \infty} \left( -\frac{1}{x^{10}} \right) = 0\]

\[\lim_{x \to \infty} \left( \frac{1}{x^{10}} \right) = 0\]

By the Squeeze Theorem, \[\lim_{x \to \infty} \frac{\cos(2x)}{x^{10}} = 0\]

(c) \[\lim_{x \to -\infty} \frac{16 \arctan(5x) + 14\pi}{4 \arctan(9x) + 5\pi}\]

Recall the graph of \( y = \arctan(x) \)

as \( x \to -\infty, 5x \to -\infty \)

and \( \arctan(5x) \to -\frac{\pi}{2} \)

Similarly, \( \arctan(9x) \to -\frac{\pi}{2} \)

\[\lim_{x \to -\infty} \frac{16 \arctan(5x) + 14\pi}{4 \arctan(9x) + 5\pi} \to \frac{16\left(-\frac{\pi}{2}\right) + 14\pi}{4\left(-\frac{\pi}{2}\right) + 5\pi} = \frac{6\pi}{3\pi} = 2\]
(d) \[ \lim_{x \to \ln 9} \frac{e^x - 9}{e^{2x} - 81} = \lim_{x \to \ln 9} \frac{e^x - 9}{(e^x)^2 - 9^2} = \lim_{x \to \ln 9} \frac{e^x - 9}{(e^x - 9)(e^x + 9)} = \lim_{x \to \ln 9} \frac{1}{e^x + 9} = \frac{1}{e^{\ln 9} + 9} = \frac{1}{9 + 9} = \frac{1}{18} \]

(e) \[ \lim_{x \to 8^+} \frac{\ln \left( \frac{1}{x^2} \right)}{1 - e^{(x^2 - 64)}} \]

\[ x \to 8^+ \Rightarrow x^2 \to 64 \Rightarrow \ln \left( \frac{1}{x^2} \right) \to \ln \left( \frac{1}{64} \right) \]

\[ x \to 8^+ \Rightarrow x^2 - 64 \to 0 \Rightarrow \text{as } e^{x^2 - 64}\text{ approaches } 1, \text{ we have } 1 - e^{x^2 - 64} \to 0^- \]

\[ \lim_{x \to 8^+} \frac{\ln \left( \frac{1}{x^2} \right)}{1 - e^{(x^2 - 64)}} \to 0^- \]

Thus \[ \lim_{x \to 8^+} \frac{\ln \left( \frac{1}{x^2} \right)}{1 - e^{(x^2 - 64)}} = 0 \]
Students – do not write on this page!

1. (12 points) ________________

2. (12 points) ________________

3. (12 points) ________________

4. (12 points) ________________

5. (12 points) ________________

6. (10 points) ________________

7. (5 points) ________________

8a. (5 points) ________________

8b. (5 points) ________________

8c. (5 points) ________________

8d. (5 points) ________________

8e. (5 points) ________________

TOTAL (100 points) ____________