Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_0^2 (6t^2 + 5e^t + 2)\ dt = \left[2t^3 + 5e^t + 2t\right]_0^2$$

$$= \left[2\cdot 2^3 + 5e^2 + 2\cdot 2\right] - \left[2\cdot 0^3 + 5e^0 + 2\cdot 0\right]$$

$$= \left[20 + 5e^2\right] - \left[5\right]$$

$$= 15 + 5e^2$$

2. (2 points each) Evaluate the following indefinite integrals.

(a) $$\int \left(\frac{6x\sqrt{x} - \sqrt{x}}{2x}\right)^2 dx = \int \left(3\sqrt{x} - \frac{\sqrt{x}}{2x}\right)^2 dx$$

$$= \int \left((3\sqrt{x})^2 - 2(3\sqrt{x})\left(\frac{\sqrt{x}}{2x}\right) + \left(\frac{\sqrt{x}}{2x}\right)^2\right) dx$$

$$= \int \left(9x - 3 + \frac{1}{4}\cdot\frac{1}{x}\right) dx$$

$$= \frac{9}{2}x^2 - 3x + \frac{1}{4}\ln|x| + C$$

(b) $$\int \frac{4\sin^5 x - 3\cos^2 x + 3}{\sin^4 x}\, dx = \int \left(\frac{4\sin^5 x + 3(1 - \cos^2 x)}{\sin^4 x}\right) dx$$

$$= \int \frac{4\sin^5 x + 3\sin^2 x}{\sin^4 x}\, dx$$

$$= \int \left(\frac{4\sin^5 x}{\sin^4 x} + \frac{3\sin^2 x}{\sin^4 x}\right) dx$$

$$= \int (4\sin x + 3\csc^2 x)\, dx$$

$$= -4\cos x - 3\cot x + C$$

3. (2 points) Suppose $H(x) = \int_{\sqrt{x}}^{10} 6\sec^9\left(t^4\right)\,dt$. Find $H'(x)$.

$$H = -\int_{10}^{\sqrt{x}} 6\sec^9(t^4)\,dt$$

Let $u = \sqrt{x}$. Then $\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$$H = -\int_{10}^{u} 6\sec^9(t^4)\,dt$$

By F.T.C. (part 1), $\frac{dH}{du} = -6\sec^9(u^4)$

$$H'(x) = \frac{dH}{dx} = \frac{dH}{du}\cdot\frac{du}{dx} \quad \text{(chain rule)}$$

$$= -6\sec^9(u^4)\cdot\frac{1}{2\sqrt{x}}$$

$$= -6\sec^9(x^2)\cdot\frac{1}{2\sqrt{x}}$$

4. (2 points) A bacteria population is 2000 at time $t = 0$. At time t hours, the population is growing at a rate of $50t + 100$ bacteria per hour. What is the population at time $t = 2$ hours?

(Pop. at $t=2$) = (Pop. at $t=0$) + (net change in pop. from $t=0$ to $t=2$)

$$= 2000 + \int_0^2 (50t+100)\,dt$$

$$= 2000 + \left[25t^2 + 100t\right]_0^2$$

$$= 2000 + [25\cdot 2^2 + 100\cdot 2] - [25\cdot 0^2 + 100\cdot 0]$$

$$= 2000 + 300$$

$$= 2300 \text{ bacteria}$$