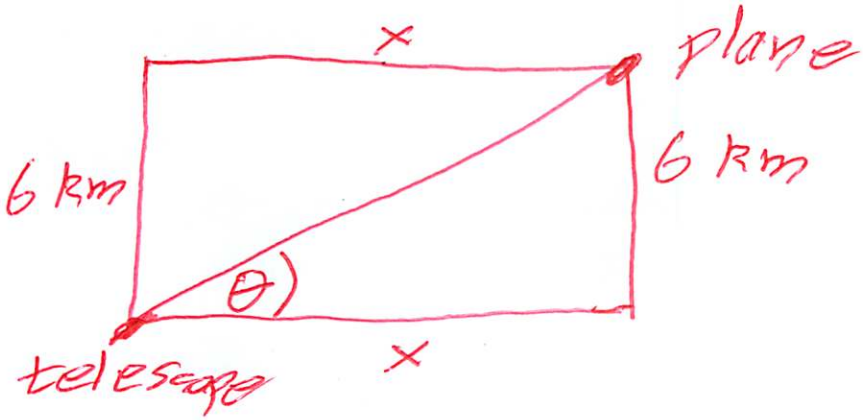
Math 220 (section AD?) **Quiz 6** **Spring 2015**

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) A plane flies horizontally at an altitude of 6 km and passes directly over a tracking telescope on the ground. When the angle of elevation is $\pi/6$ rad, this angle is decreasing at a rate of 0.5 rad/min. How fast is the plane traveling at that time?



given: $\left.\frac{d\theta}{dt}\right|_{\theta=\frac{\pi}{6}\,rad} = -0.5\ \frac{rad}{min}$

want: $\left.\frac{dx}{dt}\right|_{\theta=\frac{\pi}{6}\,rad}$

$\cot\theta = \frac{x}{6}$ $\left(\frac{adj}{opp}\right)$

$\frac{d}{dt}(\cot\theta) = \frac{d}{dt}\left(\frac{x}{6}\right)$

$-\csc^2\theta\frac{d\theta}{dt} = \frac{1}{6}\frac{dx}{dt}$

(at $\theta=\frac{\pi}{6}$, $\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\sin(\frac{\pi}{6})} = \frac{1}{1/2} = 2$

$\rightarrow -(2)^2(-0.5) = \frac{1}{6}\frac{dx}{dt}$

$\frac{dx}{dt} = 12\ km/min$

2. (4 points) A rock is thrown vertically upward from the surface of a planet. The rock's height above the planet's surface is given by the equation $s = t(24 - 1.2t)$, where t is measured in seconds and s is measured in meters.

(a) Find a formula for the rock's velocity at time t.

height: $s = t(24-1.2t)$

$= 24t - 1.2t^2$

velocity: $s' = 24 - 2.4t$

(b) What is the maximum height reached by the rock?

At its maximum height, velocity is 0,

$0 = 24 - 2.4t \Rightarrow t = 10$ seconds

max. height is $s(10) = 10(24 - 1.2(10))$

$= 120$ meters

Note: The 1st or 2nd derivative test is usually used to show this is a maximum but the physics of the problem already shows this.

3. (2 points) Solve the following differential equations given that the graph of each solution goes through the point $(2, 16)$. You must use the given variables.

(a) $\frac{dv}{dr} = 6v \Rightarrow v = Ce^{6r}$

$16 = Ce^{6\cdot 2} \Rightarrow C = \frac{16}{e^{12}}$

$v = \frac{16}{e^{12}} \cdot e^{6r}$

$= 16e^{6r-12}$

(b) $\frac{dv}{dr} = 10r \Rightarrow v = 5r^2 + C$

$16 = 5(2)^2 + C \Rightarrow C = -4$

$v = 5r^2 - 4$