Math 220 (section AD?) **Quiz 5** **Spring 2015**

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Given $g(t) = \arctan\left(t^5\right)$, find its second derivative $g''(t)$.

$$g'(t) = \frac{1}{(t^5)^2+1} \cdot \frac{d}{dt}(t^5)$$

$$= \frac{1}{t^{10}+1} \cdot 5t^4$$

$$= \frac{5t^4}{t^{10}+1}$$

$$g''(t) = \frac{\frac{d}{dt}(5t^4)\cdot(t^{10}+1) - 5t^4 \cdot \frac{d}{dt}(t^{10}+1)}{(t^{10}+1)^2}$$

$$= \frac{20t^3(t^{10}+1) - 5t^4 \cdot 10t^9}{(t^{10}+1)^2} = \frac{20t^3(t^{10}+1) - 50t^{13}}{(t^{10}+1)^2}$$

2. (2 points) Compute $w'(x)$ given that $w(x) = \sin\left(\ln\left(x^8 + 5x^2 + 2\right)\right)$.

$$w'(x) = \cos(\ln(x^8+5x^2+2)) \cdot \frac{d}{dx}(\ln(x^8+5x^2+2))$$

$$= \cos(\ln(x^8+5x^2+2)) \cdot \frac{1}{x^8+5x^2+2} \cdot \frac{d}{dx}(x^8+5x^2+2)$$

$$= \cos(\ln(x^8+5x^2+2)) \cdot \frac{1}{x^8+5x^2+2} \cdot (8x^7+10x)$$

3. (3 points) Find the equation of the line tangent to the given curve at the point $(-1, 2)$.

$$\left(2x+y^2\right)^3 = 3x^2y+2$$

$$\frac{d}{dx}\left(\left(2x+y^2\right)^3\right) = \frac{d}{dx}\left(3x^2y+2\right)$$

$$3\left(2x+y^2\right)^2 \cdot \frac{d}{dx}\left(2x+y^2\right) = \frac{d}{dx}\left(3x^2\right)\cdot y + 3x^2 \cdot \frac{d}{dx}(y) + 0$$

$$3\left(2x+y^2\right)^2 \cdot \left(2+2y\frac{dy}{dx}\right) = 6xy + 3x^2\frac{dy}{dx}$$

at $(x,y) = (-1,2)$ we have

$$3\left(2(-1)+(2)^2\right)^2 \cdot \left(2+2(2)\frac{dy}{dx}\right) = 6(-1)(2) + 3(-1)^2\frac{dy}{dx}$$

$$12 \cdot \left(2+4\frac{dy}{dx}\right) = -12 + 3\frac{dy}{dx}$$

$$24 + 48\frac{dy}{dx} = -12 + 3\frac{dy}{dx}$$

$$45\frac{dy}{dx} = -36$$

$$\frac{dy}{dx} = \frac{-36}{45} = \frac{-4}{5}$$

POINT: $(-1,2)$

SLOPE: $-4/5$

Tangent line: $y-2 = -\frac{4}{5}(x-(-1))$

$$y = -\frac{4}{5}x + \frac{6}{5}$$

4. (2 points) Compute $\frac{dy}{dx}$ for the given function. Write your answer completely in terms of x.

$$y = \left(x^4 + 2\right)^{\tan x}$$

Method 1

$$\ln(y) = \ln\left((x^4+2)^{\tan x}\right)$$

$$\ln(y) = \tan(x) \cdot \ln(x^4+2)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}\left(\tan(x) \cdot \ln(x^4+2)\right)$$

$$\frac{1}{y}\frac{dy}{dx} = \sec^2(x) \cdot \ln(x^4+2) + \tan(x) \cdot \frac{1}{x^4+2} \cdot 4x^3$$

$$\frac{dy}{dx} = y\left(\sec^2(x)\ln(x^4+2) + \frac{4x^3\tan(x)}{x^4+2}\right)$$

$$\frac{dy}{dx} = (x^4+2)^{\tan(x)} \cdot \left(\sec^2(x)\ln(x^4+2) + \frac{4x^3\tan(x)}{x^4+2}\right)$$

Method 2

$$y = (x^4+2)^{\tan x}$$

$$= e^{\ln\left((x^4+2)^{\tan x}\right)}$$

$$= e^{\tan x \cdot \ln(x^4+2)}$$

$$\frac{dy}{dx} = e^{\tan x \cdot \ln(x^4+2)} \cdot \frac{d}{dx}\left(\tan x \cdot \ln(x^4+2)\right)$$

$$\frac{dy}{dx} = e^{\tan x \cdot \ln(x^4+2)} \cdot \left(\sec^2 x \ln(x^4+2) + \tan x \cdot \frac{4x^3}{x^4+2}\right)$$