1. (2 points) What is the slope of the line tangent to the graph of \( g(x) \) at its y-intercept? Simplify your answer.

\[ g(x) = 40 + 3e^x + 5\sin x \]

\[ g'(x) = 3e^x + 5\cos(x) \]

\( x = 0 \) at the y-intercept, so the slope is \( g'(0) = 8 \)

2. (2 points) Explain carefully why there is no line which is both tangent to the graph of \( f(x) \) and parallel to the line \( 10x + 2y = 9 \).

\[ f(x) = 3e^x + 2x^3 - 4x - 8 \]

\[ 10x + 2y = 9 \Rightarrow y = -5x + \frac{9}{2} \] which has slope -5.

Each line parallel to this line must also have slope -5.

Since \( f'(x) = 3e^x + 6x^2 - 4 > -4 \) for all \( x \), it is impossible for any tangent line to be parallel to the given line.
3. (2 points each) Using Leibniz notation (i.e., \( \frac{dy}{dx}, \frac{dp}{dt} \), etc.), find derivatives for each of the following functions.

(a) \( \theta = \left( t^2 \sqrt{t} \right)^6 \) (simplify your answer)

\[
\theta = (t^2 \cdot t^{1/2})^6 \\
= (t^2)^6 \cdot (t^{1/2})^6 \\
= (t^2)^6 \cdot (t^{3\cdot6}) \\
= t^{12} \cdot t^2 \\
= t^{14}
\]

\( \frac{d\theta}{dt} = 14t^{13} \)

(b) \( R = x^3 \sec x + 2e^5 \)

\[
\frac{dR}{dx} = \frac{d}{dx} (x^3 \cdot \sec x) + x^3 \cdot \frac{d}{dx} (\sec x) \\
\frac{dR}{dx} = 3x^2 \sec (x) + x^3 \sec (x) \tan (x)
\]

Note: \( \frac{dx}{dx} (2e^5) = 0 \) since \( 2e^5 \) is a constant

(c) \( V = \frac{e^z + \tan z}{z^5 + 8z} \)

\[
\frac{dV}{dz} = \frac{d}{dz} \left( \frac{e^z + \tan (z)}{z^5 + 8z} \right) \\
\frac{dV}{dz} = \frac{(e^z + \tan (z)) \cdot (z^5 + 8z) - (e^z + \tan (z)) \cdot (5z^4 + 8)}{(z^5 + 8z)^2}
\]

\[
\frac{dV}{dz} = \frac{(e^z + \sec^2 (z)) \cdot (z^5 + 8z) - (e^z + \tan (z)) \cdot (5z^4 + 8)}{(z^5 + 8z)^2}
\]