

Name

Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) What is the slope of the line tangent to the graph of $g(x)$ at its y -intercept? Simplify your answer.

$$g(x) = 40 + 3e^x + 5 \sin x$$

$$g'(x) = 3e^x + 5 \cos(x)$$

$x=0$ at the y -intercept,
so the slope is $g'(0) = 8$

2. (2 points) Explain carefully why there is no line which is both tangent to the graph of $f(x)$ and parallel to the line $10x + 2y = 9$.

$$f(x) = 3e^x + 2x^3 - 4x - 8$$

$10x + 2y = 9 \Rightarrow y = -5x + \frac{9}{2}$ which has slope -5 .

Each line parallel to this line must also have slope -5 .

Since $f'(x) = 3e^x + 6x^2 - 4 > -4$ for all x , it is impossible for any tangent line to ~~be~~ be parallel to the given line.

3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $\theta = (t^2 \sqrt[3]{t})^6$ (simplify your answer)

$$\begin{aligned}\theta &= (t^2 \cdot t^{1/3})^6 \\ &= (t^2)^6 \cdot (t^{1/3})^6 \\ &= (t^{2 \cdot 6}) \cdot (t^{1/3 \cdot 6}) \\ &= t^{12} \cdot t^2 \\ &= t^{14}\end{aligned}$$

$$\frac{d\theta}{dt} = 14t^{13}$$

(b) $R = x^3 \sec x + 2e^5$

$$\frac{dR}{dx} = \frac{d}{dx}(x^3) \cdot \sec(x) + x^3 \cdot \frac{d}{dx}(\sec(x))$$

$$\frac{dR}{dx} = 3x^2 \sec(x) + x^3 \sec(x) \tan(x)$$

Note: $\frac{d}{dx}(2e^5) = 0$ since $2e^5$ is a constant

(c) $V = \frac{e^z + \tan z}{z^5 + 8z}$

$$\frac{dV}{dz} = \frac{\frac{d}{dz}(e^z + \tan(z)) \cdot (z^5 + 8z) - (e^z + \tan(z)) \cdot \frac{d}{dz}(z^5 + 8z)}{(z^5 + 8z)^2}$$

$$\frac{dV}{dz} = \frac{(e^z + \sec^2(z))(z^5 + 8z) - (e^z + \tan(z)) \cdot (5z^4 + 8)}{(z^5 + 8z)^2}$$