Math 220 (section AD?) Quiz 3 Spring 2015

Name Solutions

- 20 minutes
- No calculators
- Show sufficient work
- Do not use derivatives

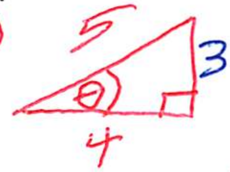
1. (2 points) Evaluate $\csc(2\arccos(4/5))$.

(1) Let $\theta = \arccos(4/5)$

Then $\cos(\theta) = 4/5$ with $0 \le \theta \le \pi$

(actually $0 \le \theta \le \pi/2$ since $\cos(\theta) \ge 0$)

(2)



$\cos(\theta) = \frac{4}{5} \left(\frac{\text{adj}}{\text{hyp}}\right)$

From Pyth. Thm,

$4^2 + (\text{opp})^2 = 5^2$

$\Rightarrow \text{opp} = 3$

(3)

$\csc(2\arccos(\tfrac{4}{5})) =$

$\csc(2\theta) =$

$\frac{1}{\sin(2\theta)} =$

$\frac{1}{2\sin(\theta)\cos(\theta)} =$

$\frac{1}{2(\frac{3}{5})(\frac{4}{5})} =$

$\frac{25}{24}$

2. (1 point) Which one of the following equations must hold in order for a function w to be continuous at a number p ?

(a) $\lim_{x\to 0} w(x) = w(p)$

(b) $\lim_{x\to 0} w(x) = 0$

(c) $\lim_{x\to 0} w(x) = p$

(d) $\lim_{x\to p} w(x) = w(p)$

(e) $\lim_{x\to p} w(x) = 0$

(f) $\lim_{x\to p} w(x) = p$

(g) $\lim_{x\to\infty} w(x) = w(p)$

(h) $\lim_{x\to\infty} w(x) = 0$

(i) $\lim_{x\to\infty} w(x) = p$

3. (2 points each) Evaluate the following limits. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is ∞ or $-\infty$.

(a) $\lim_{x \to 4^-} \frac{\sqrt[3]{x-12}}{x-4}$ $\xrightarrow{} -2$, $\xrightarrow{} 0^-$ $= \infty$

$\sqrt[3]{4-12} = \sqrt[3]{-8} = -2$

(note: The form $\frac{\text{nonzero}}{\text{zero}}$ gave an infinite limit and $\frac{\text{neg}}{\text{neg}} = \text{pos}.$)

(b) $\lim_{x \to 0} \left(\frac{1}{2x} - \frac{3}{x^2+6x} \right) = \lim_{x \to 0} \left(\frac{1}{2x} - \frac{3}{x(x+6)} \right)$

$= \lim_{x \to 0} \left(\frac{1}{2x} \cdot \frac{x+6}{x+6} - \frac{3}{x(x+6)} \cdot \frac{2}{2} \right)$

$= \lim_{x \to 0} \frac{x+6-6}{2x(x+6)}$

$= \lim_{x \to 0} \frac{x}{2x(x+6)}$

$= \lim_{x \to 0} \frac{1}{2(x+6)}$

$= \frac{1}{2(0+6)}$

$= \frac{1}{12}$

4. (3 points) Find all horizontal asymptotes on the graph of $f(x) = \dfrac{16+15e^{2x}}{3e^{2x}-8}$.

$$\lim_{x\to\infty} \frac{16+15e^{2x} \nearrow \infty}{3e^{2x}-8 \to \infty} = \lim_{x\to\infty} \frac{16+15e^{2x}}{3e^{2x}-8} \cdot \frac{1/e^{2x}}{1/e^{2x}}$$

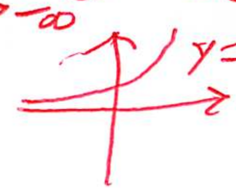
$$= \lim_{x\to\infty} \frac{16/e^{2x}+15}{3-8/e^{2x}}$$

$$= \frac{0+15}{3-0}$$

$$= 5$$

$$\lim_{x\to-\infty} \frac{16+15e^{2x} \nearrow 16}{3e^{2x}-8 \to -8} = \frac{16}{-8} = -2$$

(note: $\lim_{x\to-\infty} e^{2x} = 0$



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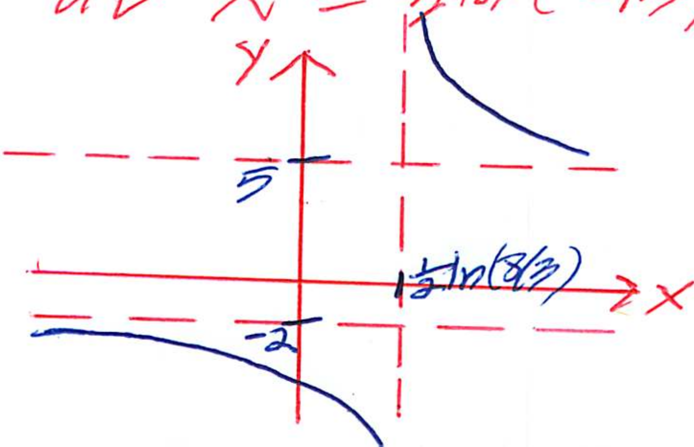
$f(x) = \dfrac{16+15e^{2x}}{3e^{2x}-8}$ has horizontal asymptotes at $y=5$ and $y=-2$

note: $f(x)$ also has a vertical asymptote at $x = \frac{1}{2}\ln(8/3)$



$f(x) = \dfrac{16+15e^{2x}}{3e^{2x}-8}$