Math 220 (section AD?) Quiz 11 Spring 2015

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) Find the average value of the function $f(x) = \dfrac{20x}{(x^2+1)^2}$ on the interval $[1, 3]$. Simplify your answer.

$$f_{ave} = \frac{1}{3-1}\int_1^3 \frac{20x}{(x^2+1)^2}\,dx$$

$$= \frac{1}{2}\int_2^{10} \frac{10\,du}{u^2}$$

$$= \frac{10}{2}\int_2^{10} u^{-2}\,du$$

$$= 5\left[-u^{-1}\right]_2^{10}$$

$$= 5\left[\frac{-1}{10} - \left(\frac{-1}{2}\right)\right]$$

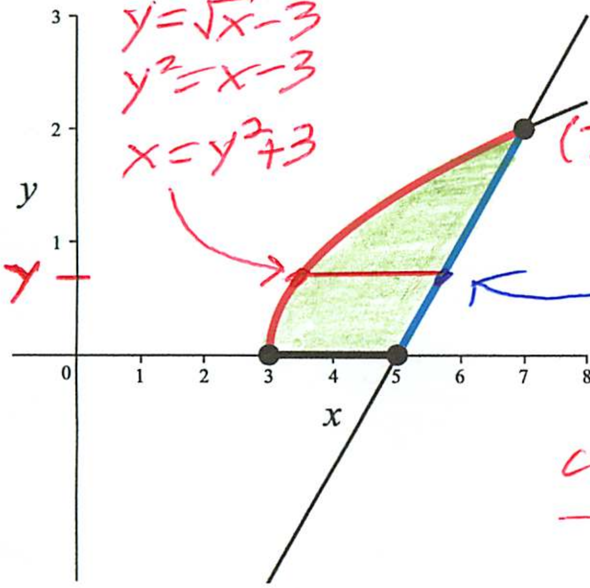
$$= 2$$

$$\left(\begin{array}{l} u = x^2+1 \\ du = 2x\,dx \\ 10\,du = 20x\,dx \\ x=1 \Rightarrow u = 1^2+1 = 2 \\ x=3 \Rightarrow u = 3^2+1 = 10 \end{array}\right)$$

2. Let **R** be the finite region bounded by the graphs of $y = \sqrt{x-3}$, $y = x - 5$ and $y = 0$. Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

(a) (3 points) The volume of the solid with base **R** for which the cross-sections perpendicular to the y-axis are squares.

intersection: $x - 5 = \sqrt{x-3}$
$x^2 - 10x + 25 = x - 3$
$x^2 - 11x + 28 = 0$
$(x-7)(x-4) = 0$
$\to x = 7$, $\cancel{x=4}$
point: $(7,2)$



$y = \sqrt{x-3}$
$y^2 = x - 3$
$x = y^2 + 3$

$(7,2)$

$y = x - 5$
$x = y + 5$

y

cross-section at slice at y

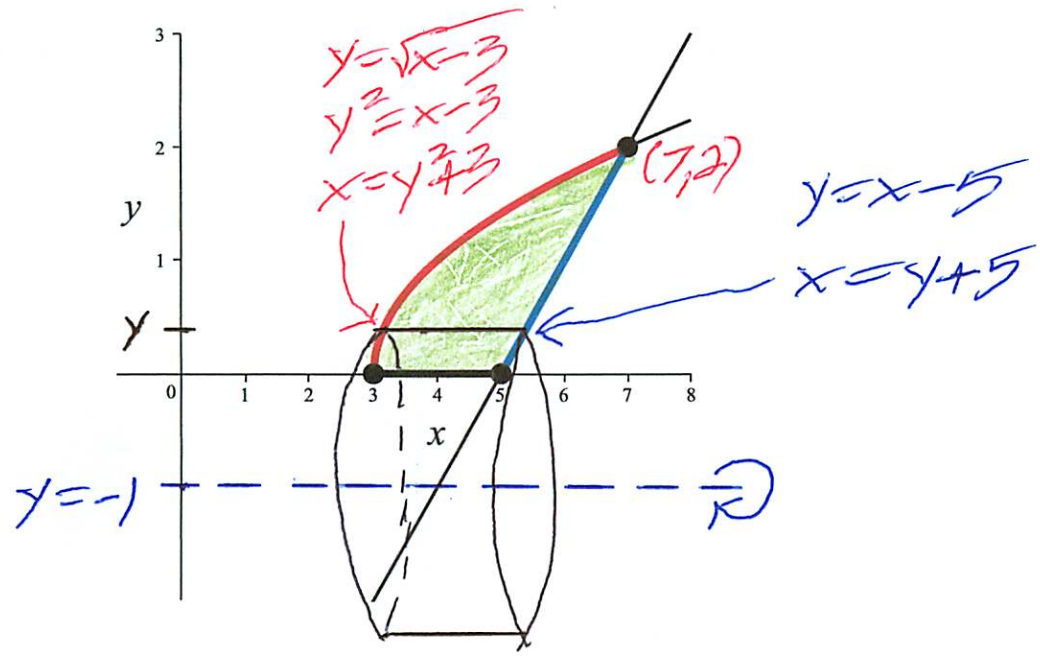
area = (side length)^2

$$V = \int_0^2 (\text{cross-sectional area})\, dy$$

$$= \int_0^2 (\text{side length})^2\, dy$$

$$= \int_0^2 (y + 5 - (y^2 + 3))^2\, dy$$

ii. (2 points) Integrate with respect to y. (Use different integrands in parts i and ii.)



When a horizontal slice at y is revolved around a horizontal line, it gives a cylinder with surface area $2\pi rh$.

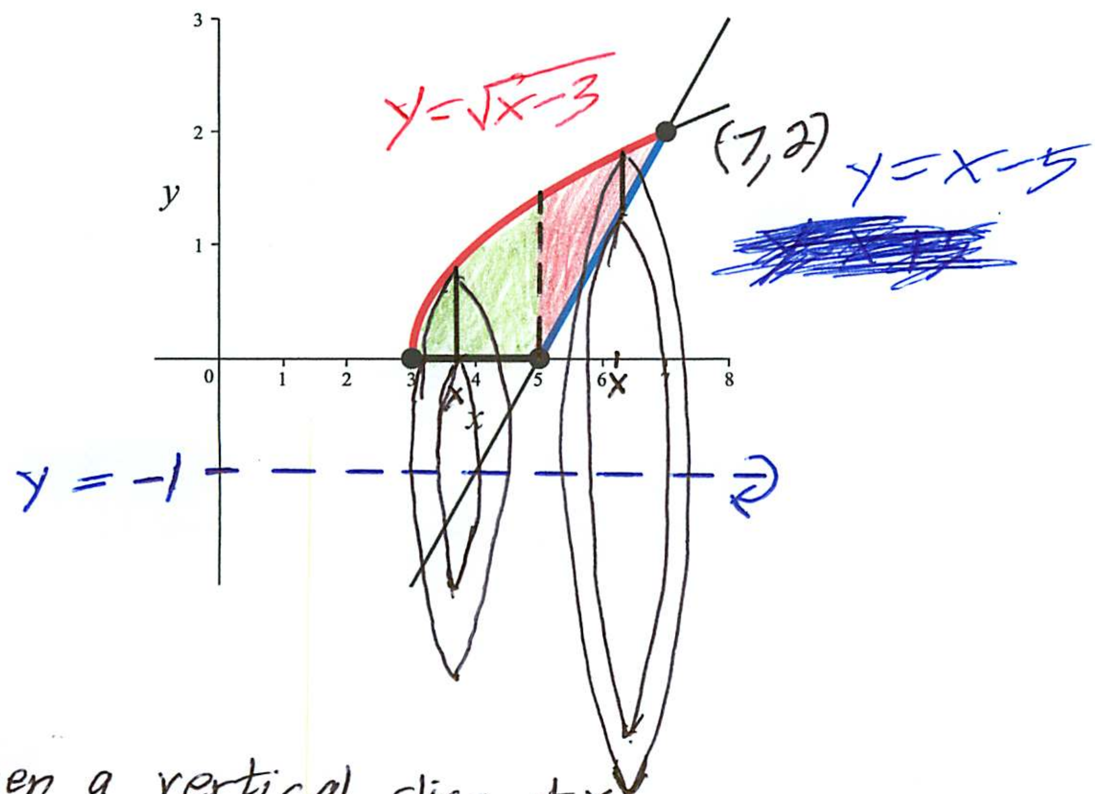
$$V=\int_0^2 (\text{surface area})\,dy$$

$$=\int_0^2 2\pi\cdot\text{radius}\cdot\text{height}\,dy$$

$$=\int_0^2 2\pi\,(y-(-1))\,(y+5-(y^2+3))\,dy$$

(b) The volume of the solid formed when **R** is revolved around the line $y = -1$. Determine this volume in the following two ways.

i. (2 points) Integrate with respect to x.



when a vertical slice at x is revolved around a horizontal line, it gives a cross-section with area $\pi \cdot r_{out}^2 - \pi \cdot r_{in}^2$

$$V = \int_{x_{min}}^{x_{max}} (\text{cross-sectional area})\,dx$$

$$V = \int_3^5 \left(\pi\left(\sqrt{x-3}-(-1)\right)^2 - \pi\left(0-(-1)\right)^2\right)dx$$

$$+ \int_5^7 \left(\pi\left(\sqrt{x-3}-(-1)\right)^2 - \pi\left(x-5-(-1)\right)^2\right)dx$$