Math 220 (section AD?) **Quiz 10** **Spring 2015**

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

suppose f satisfies these two conditions.

① f is continuous on [a, b]

② f is differentiable on (a, b)

Then there is a c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_{e^4}^{e^{25}} \frac{7}{x\sqrt{\ln x}}\,dx$$

Method 1

let $u = \ln x$

$du = \frac{1}{x}dx$

$x = e^4 \Rightarrow u = \ln(e^4) = 4$

$x = e^{25} \Rightarrow u = \ln(e^{25}) = 25$

$$\int_{e^4}^{e^{25}} \frac{7}{x\sqrt{\ln x}}dx = \int_4^{25} \frac{7}{\sqrt{u}}du = \int_4^{25} 7u^{-1/2}du$$

$$= \left[14u^{1/2}\right]_4^{25} = 14\sqrt{25} - 14\sqrt{4} = 70 - 28 = 42$$

Method 2

let $u = \sqrt{\ln x}$

$du = \frac{1}{2}(\ln x)^{-1/2} \cdot \frac{1}{x}dx$

$2du = \frac{1}{x\sqrt{\ln x}}dx$

$x = e^4 \Rightarrow u = \sqrt{\ln(e^4)} = 2$

$x = e^{25} \Rightarrow u = \sqrt{\ln(e^{25})} = 5$

$$\int_{e^4}^{e^{25}} \frac{7}{x\sqrt{\ln x}}dx = \int_2^5 7 \cdot 2du = \left[14u\right]_2^5$$

$= 14 \cdot 5 - 14 \cdot 2$

$= 70 - 28$

$= 42$

3. (2 points each) Evaluate the indefinite integrals.

(a) $\int \frac{10x^3 + 16x}{x^2+1}\,dx$

$$\begin{array}{r} 10x \\ x^2+1 \overline{) \, 10x^3+16x} \\ 10x^3+10x \\ \hline 6x \end{array}$$

Method 1

use polynomial long division to get

$$\int \frac{10x^3+16x}{x^2+1}\,dx = \int \left(10x + \frac{6x}{x^2+1}\right)dx$$

$$= \int 10x\,dx + \int \frac{6x}{x^2+1}\,dx$$

$$= 5x^2 + 3\ln(x^2+1) + C$$

$u = x^2+1$
$du = 2x\,dx$
$3du = 6x\,dx$
$\int \frac{6x}{x^2+1}dx = \int \frac{3du}{u}$
$= 3\int \frac{1}{u}du = 3\ln|u| + C$
$= 3\ln(x^2+1) + C$

Method 2

$u = x^2+1$
$du = 2x\,dx$

$$\int \frac{10x^3+16x}{x^2+1}\,dx = \int \frac{2x(5x^2+8)}{x^2+1}\,dx = \int \frac{5x^2+5+3}{x^2+1}\cdot 2x\,dx$$

$$= \int \frac{5(x^2+1)+3}{x^2+1}\cdot 2x\,dx$$

$$= \int \frac{5u+3}{u}\,du = \int \left(\frac{5u}{u} + \frac{3}{u}\right)du$$

$$= \int \left(5 + 3\cdot\frac{1}{u}\right)du = 5u + 3\ln|u| + C$$

$$= 5(x^2+1) + 3\ln(x^2+1) + C$$

(b) $\int 6x^5\left(x^3+1\right)^{10}\,dx$

$u = x^3+1$ $\quad$ (so $x^3 = u-1$)

$du = 3x^2\,dx$

$$\int 6x^5(x^3+1)^{10}\,dx = \int 2x^3(x^3+1)^{10}\,3x^2\,dx$$

$$= \int 2(u-1)u^{10}\,du$$

$$= \int (2u^{11} - 2u^{10})\,du$$

$$= 2\cdot\tfrac{1}{12}u^{12} - 2\cdot\tfrac{1}{11}u^{11} + C$$

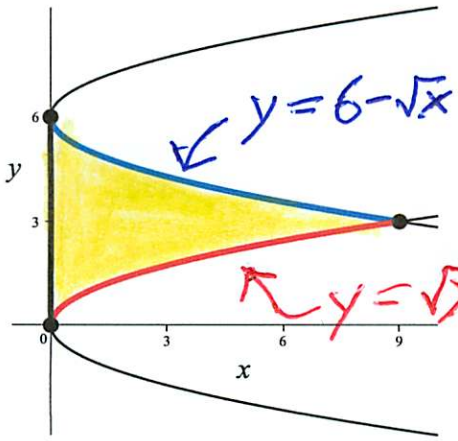
$$= \frac{1}{6}(x^3+1)^{12} - \frac{2}{11}(x^3+1)^{11} + C$$

4. (2 points) Let **R** be the finite region bounded by $x = y^2$, $x = (y-6)^2$, and $x = 0$. In the following manner, set up <u>but do not evaluate</u> definite integrals which represent the area of the region **R**.

(a) Integrate with respect to x.

$x = y^2 \Rightarrow y = \pm\sqrt{x}$

$x = (y-6)^2 \Rightarrow y = 6 \pm \sqrt{x}$



intersection

$y^2 = (y-6)^2$

$y^2 = y^2 - 12y + 36$

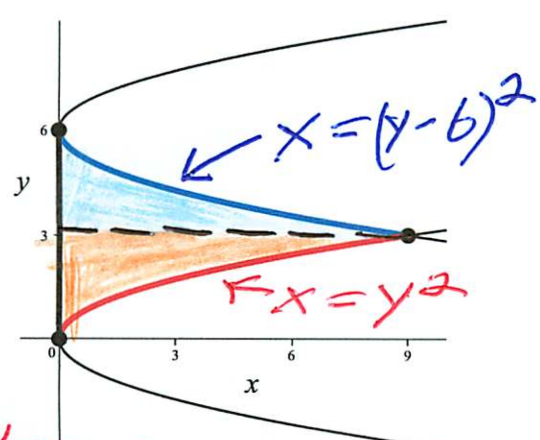
$0 = -12y + 36$

$y = 3$ (so $x = 3^2 = 9$)

$$A = \int_{x_{min}}^{x_{max}} (y_{top} - y_{bottom})\,dx$$

$$= \int_0^9 \left((6 - \sqrt{x}) - \sqrt{x}\right)dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)



$$A = \int_{y_{min}}^{y_{max}} (x_{right} - x_{left})\,dy$$

$$= \int_0^3 (y^2 - 0)\,dy + \int_3^6 ((y-6)^2 - 0)\,dy$$