Math 220 (section AD?) **Quiz 9** **Spring 2014**

Name Solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Suppose $w(x) = \int_{x^2}^{5} \tan^6 t\, dt$. Find $w'(x)$.

Let $u = x^2$ $\quad w = \int_u^5 \tan^6 t\, dt = -\int_5^u \tan^6 t\, dt$

$\frac{du}{dx} = 2x \quad \frac{dw}{du} = -\tan^6 u$ by Fund. Thm. of Calculus (Part 1)

$w'(x) = \frac{dw}{dx} = \underbrace{\frac{dw}{du} \cdot \frac{du}{dx}}_{\text{(chain rule)}} = -\tan^6 u \cdot 2x = -\tan^6(x^2) \cdot 2x$

2. (2 points) Evaluate the following indefinite integral.

$$\int \frac{(x^2+3)^2}{x^3}\, dx = \int \frac{x^4+6x^2+9}{x^3} dx = \int \left(\frac{x^4}{x^3} + \frac{6x^2}{x^3} + \frac{9}{x^3}\right) dx$$

$$= \int \left(x + \frac{6}{x} + 9x^{-3}\right) dx$$

$$= \frac{1}{2}x^2 + 6\ln|x| + 9 \cdot \frac{1}{-2} x^{-2} + C$$

$$= \frac{1}{2}x^2 + 6\ln|x| - \frac{9}{2}x^{-2} + C$$

3. (2 points) Evaluate the following indefinite integral.

$$\int \frac{\cot^2(\theta)+1}{\cot^2(\theta)}\, d\theta = \int \left(\frac{\cot^2\theta}{\cot^2\theta} + \frac{1}{\cot^2\theta}\right) d\theta$$

$$= \int (1 + \tan^2\theta)\, d\theta$$

$$= \int \sec^2\theta\, d\theta$$

$$= \tan\theta + C$$

4. (2 points) Evaluate the following definite integral. Simplify your answer.

$$\int_1^4 \frac{18\sqrt{x}-x^3}{3x}\,dx = \int_1^4 \left(\frac{18\sqrt{x}}{3x} - \frac{x^3}{3x}\right)dx$$

$$= \int_1^4 \left(6x^{-1/2} - \frac{1}{3}x^2\right)dx$$

$$= \left(6\cdot\frac{1}{1/2}x^{1/2} - \frac{1}{3}\cdot\frac{1}{3}x^3\right)\Big|_1^4$$

$$= \left(12\sqrt{x} - \frac{1}{9}x^3\right)\Big|_1^4$$

$$= \left(12\sqrt{4} - \frac{1}{9}(4)^3\right) - \left(12\sqrt{1} - \frac{1}{9}(1)^3\right)$$

$$= \left(24 - \frac{64}{9}\right) - \left(12 - \frac{1}{9}\right)$$

$$= 24 - 12 - \frac{64}{9} + \frac{1}{9}$$

$$= 12 - \frac{63}{9} = 12 - 7 = 5$$

---

5. (2 points) The height of a tree is currently 10 ft. Suppose the tree's height is increasing by $6\sqrt{t}$ in/month where t is measured in months from now. What will the tree's height be in 4 months? (Note: 1 ft = 12 in)

$= 120$ in

height in 4 months = current height + net change in height between $t=0$ and $t=4$ mon.

$$= 120 + \int_0^4 \left(\text{rate of change of height}\right)dt$$

$$= 120 + \int_0^4 6\sqrt{t}\,dt$$

$$= 120 + \left(6\cdot\frac{1}{3/2}t^{3/2}\right)\Big|_0^4$$

$$= 120 + \left(4t^{3/2}\right)\Big|_0^4$$

$$= 120 + \left(4(4)^{3/2} - 4(0)^{3/2}\right)$$

$$= 120 + 32 = 152 \text{ inches}$$