Name.SOLUTIONS.

(circle your TA discussion section)

- AD1, TR 9:00-10:50, Darlayne Addabbo
- AD2, TR 1:00-2:50, Ben Fulan
- ADA, TR 8:00-8:50, Chris Bailey
- ADB, TR 9:00-9:50, Chris Bailey
- ADC, TR 10:00-10:50, Andrew McConvey
- ADD, TR 11:00-11:50, Dina Taha
- ADE, TR 12:00-12:50, Paul Spiegelhalter
- ADF, TR 1:00-1:50, Dina Taha
- ADG, TR 2:00-2:50, Paulina Koutsaki
- ADH, TR 3:00-3:50, Paulina Koutsaki
- ADJ, TR 9:00-9:50, Jed Chou
- ADK, TR 10:00-10:50, Jed Chou
- ADL, TR 11:00-11:50, Andrew McConvey
- ADM, TR 12:00-12:50, Benjamin Wright
- ADN, TR 1:00-1:50, Benjamin Wright
- ADQ, TR 2:00-2:50, Paul Spiegelhalter
- ADP, TR 3:00-3:50, Wau-Yu Wu
- ADQ, TR 4:00-4:50, Wau-Yu Wu

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.

- You may use your notes, the textbook, or information found on my course home page.

- You may use a calculator only for basic arithmetic. In particular you should not use its graphing features.

- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.

- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, March 14.

- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.

- Be sure that the pages are nicely stapled – do not just fold the corners.

- Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.
1. (2 points) Evaluate the following limit.

\[ \lim_{x \to 0} (1 - 8x)^{9/x} \]

This limit is of indeterminate form \( \infty \) as \( x \to 0^+ \) and \( -\infty \) as \( x \to 0^- \).

\[ \lim_{x \to 0} (1 - 8x)^{9/x} = \lim_{x \to 0} e^{\ln ((1 - 8x)^{9/x})} \]

\[ = e^{\lim_{x \to 0} \ln ((1 - 8x)^{9/x})} \]

\[ = e^{\lim_{x \to 0} \frac{9}{x} \ln (1 - 8x)} \]

\[ = e^{\lim_{x \to 0} \frac{9 \ln (1 - 8x)}{x}} \]

\[ = e^{\lim_{x \to 0} \frac{\frac{9}{1 - 8x} \cdot (-8)}{1}} \]

\[ = e^{\lim_{x \to 0} \left( \frac{-72}{1 - 8x} \right)} \]

\[ = e^{-72} \]

This is okay since \( e^y \) is continuous at all \( y \) by l'Hopital's Rule.
2. (3 points) Find the absolute maximum and absolute minimum values of the given function on the interval \([-4, 1]\).

\[ f(x) = \frac{-2x}{(x^2 + 9)^{5/2}} \]

\[ f'(x) = \frac{(-2)(x^2 + 9)^{3/2} - (-2x)(\frac{3}{2})(x^2 + 9)^{3/2}(2x)}{(x^2 + 9)^5} \]

\[ = \frac{2(x^2 + 9)^{3/2}(-x^3 - 9 + 5x^2)}{(x^2 + 9)^5} \]

\[ = \frac{2(4x^2 - 9)}{(x^2 + 9)^{7/2}} = \frac{2(2x - 3)(2x + 3)}{(x^2 + 9)^{7/2}} \]

\( f' \) exists for all \( x \) so the only critical numbers for \( f \) are \( x = \frac{3}{2} \) and \( x = -\frac{3}{2} \) where \( f' = 0 \). \( \frac{3}{2} \) is not in the interval \([-4, 1]\].

Since \( f \) is continuous on \([-4, 1]\) we use the closed interval method.

\[ f(-4) = \frac{-2(-4)}{((-4)^2 + 9)^{3/2}} = \frac{8}{3125} = 0.00256 \]

\[ f(-\frac{3}{2}) = \frac{-2(-\frac{3}{2})}{((-\frac{3}{2})^2 + 9)^{3/2}} = \frac{32\sqrt{3}}{10125} \approx 0.00707 \text{ (abs max)} \]

\[ f(1) = \frac{-2(1)}{(1^2 + 9)^{3/2}} = -\frac{30}{500} = -0.00632 \text{ (abs min)} \]

The abs. max of \( f(x) \) on \([-4, 1]\) is \( f(-\frac{3}{2}) = \frac{32\sqrt{3}}{10125} \)

The abs. min of \( f(x) \) on \([-4, 1]\) is \( f(1) = -\frac{30}{500} \)
$$f(x) = \frac{-2x}{(x^2+9)^{3/2}}$$
on $[-4, 1]$ in red
3. (2 points) Find each interval of concavity and each inflection point for the given function.

\[ f(x) = \frac{x - 5}{x^3} \]

\[ f'(x) = \frac{(1)(x^3) - (x-5)(3x^2)}{(x^3)^2} \]

\[ = \frac{x^2(x - (x-5)3)}{x^6} \]

\[ = \frac{15 - 2x}{x^4} \]

\[ f''(x) = \frac{(-2)(x^4) - (15-2x)(4x^3)}{(x^4)^2} \]

\[ = \frac{x^3(-2x - (15-2x)4)}{x^8} \]

\[ = \frac{6x - 60}{x^5} \]

\[ = \frac{6(x - 10)}{x^5} \]

\[ \text{values of } f''(x) \]

\[ \begin{array}{ccc}
++ & \text{und} & +++ \\
0 & 10 & \end{array} \]

\[ f \text{ is concave up on } (-\infty, 0) \text{ since } f'' > 0 \]

\[ f \text{ is concave down on } (0, 10) \text{ since } f'' < 0 \]

\[ f \text{ is concave up on } (10, \infty) \text{ since } f'' > 0 \]

\[ x=0 \text{ is not in the domain of } f(x), \text{ so } (0, f(0)) \text{ is not an inflection point of } f. \]

However \( (10, f(10)) = (10, \frac{1}{300}) \) is an inflection point of \( f \) since \( f \) switches concavity at \( x=10 \).
\[ f(x) = \frac{x-5}{x^3} \]
4. (3 points) For each $x > 0$, a triangle is formed with vertices $(0,0)$, $(x,7e^{-6x})$ and $(x,-2e^{-6x})$. What is the value of $x$ which results in the triangle of largest area?

\[
\text{area } A = \frac{1}{2} (x)(9e^{-6x}) \\
= \frac{9}{2}xe^{-6x} \quad \text{(maximize } A \text{ on } (0, \infty))
\]

\[
A' = \frac{9}{2}(e^{-6x}) + \frac{9}{2}x(-6e^{-6x}) \\
= \frac{9}{2}e^{-6x}(1-6x)
\]

\[A' = 0 \text{ at } x = \frac{1}{6}\]

The maximum area occurs at $x = \frac{1}{6}$, where area is

\[
A = \frac{9}{2} \left(\frac{1}{6}\right) e^{-6\left(\frac{1}{6}\right)} \\
= \frac{9}{12} e = \frac{3}{4e}
\]