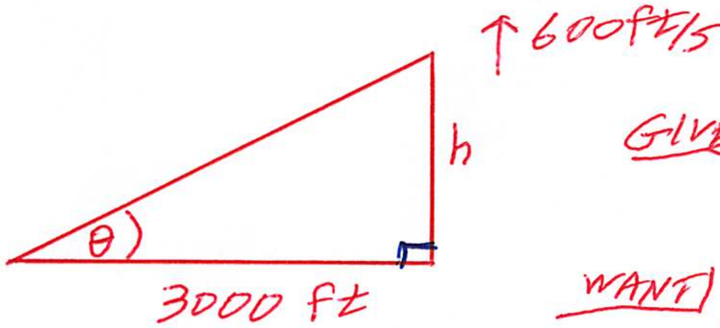
Math 220 (section AD?) Quiz 6 Spring 2014

Name Solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A camera is positioned on the ground 3000 feet from the base of a rocket launching pad. The angle of elevation of the camera has to change at the correct rate in order to keep the rocket in sight. Suppose the rocket rises vertically and its speed is 600 feet per second when it has risen 4000 feet. How quickly is the camera's angle of elevation increasing at that moment?



GIVEN $\left.\frac{dh}{dt}\right|_{h=4000\text{ ft}} = 600 \text{ ft/s}$

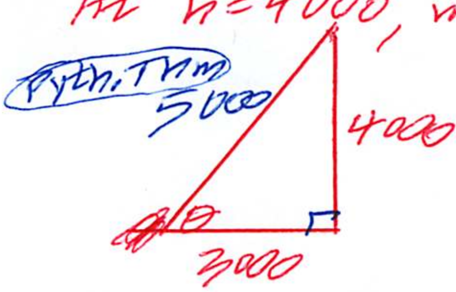
WANT $\left.\frac{d\theta}{dt}\right|_{h=4000\text{ ft}}$

$\tan\theta = \frac{h}{3000}$

$\frac{d}{dt}(\tan\theta) = \frac{d}{dt}\left(\frac{h}{3000}\right)$

$\sec^2\theta \frac{d\theta}{dt} = \frac{1}{3000}\frac{dh}{dt}$

At $h=4000$, we have



Thus $\sec\theta = \frac{5000}{3000} = \frac{5}{3}$ so

$\left(\frac{5}{3}\right)^2 \frac{d\theta}{dt} = \frac{1}{3000}\cdot 600$

$\frac{d\theta}{dt} = \frac{600}{3000}\cdot\frac{9}{25}$
$= \frac{9}{125}$ rad/s

2. (3 points) Determine a formula for w as a function of r given that $6w' + 3w = 0$ and $w(0) = 3$. Hint: You may recognize the solution more quickly if you first solve the given equation for w'.

$6w' + 3w = 0 \Rightarrow 6w' = -3w \Rightarrow w' = \frac{1}{2}w$

$\frac{dw}{dr} = -\frac{1}{2}w \Rightarrow w = ce^{-\frac{1}{2}r}$

$w(0) = 3 \Rightarrow 3 = ce^{-\frac{1}{2}\cdot 0} \Rightarrow c = 3$

$w = 3e^{-\frac{1}{2}r}$

3. (4 points) A man is standing on a bridge over a river. He reaches over the railing and throws a stone vertically upward. Until it lands in the river, the stone's height in feet above the river is $h = -16t^2 + 64t + 80$ where t is measured in seconds since the stone was thrown.

(a) What is the maximum height reached by the stone?

(height) $h = -16t^2 + 64t + 80$

(velocity) $h' = -32t + 64$

at max. height, $h' = 0$

$0 = -32t + 64 \Rightarrow t = 2$ sec.

$h(2) = -16(2)^2 + 64(2) + 80$
$= 144$ ft

note: We know physically that it is a max. when $h' = 0$. We can prove it with calculus using derivatives.

values of h': $+ +$ 0 $- -$ at $t = 2$

(b) What is the velocity of the stone as it strikes the river?

when it strikes the river, $h = 0$

$0 = -16t^2 + 64t + 80$

$= -16(t^2 - 4t - 5)$

$= -16(t+1)(t-5)$

$t = -1$ or $t = 5$

not valid since $t \geq 0$ in our problem

$h'(5) = -32(5) + 64$
$= -96$ ft/s