Math 220 (section AD?) Quiz 5 Spring 2014

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) Given $g(t) = \cos\left(t^5\right)$, find its second derivative $g''(t)$.

$g'(t) = -\sin(t^5) \cdot \frac{d}{dt}(t^5) = -\sin(t^5) \cdot 5t^4$

$g'(t) = -5t^4 \sin(t^5)$

$g''(t) = \frac{d}{dt}(-5t^4) \cdot \sin(t^5) + (-5t^4) \cdot \frac{d}{dt}(\sin(t^5))$

$= -20t^3 \sin(t^5) - 5t^4 \cos(t^5) 5t^4$

$= -20t^3 \sin(t^5) - 25t^8 \cos(t^5)$

2. (2 points each) Compute $\frac{dy}{dx}$ given each of the following equations.

(a) $y = \sqrt{\arcsin\left(e^{-5x}\right)}$ $= (\arcsin(e^{-5x}))^{1/2}$

$\frac{dy}{dx} = \frac{1}{2}(\arcsin(e^{-5x}))^{-1/2} \cdot \frac{d}{dx}(\arcsin(e^{-5x}))$

$= \frac{1}{2}(\arcsin(e^{-5x}))^{-1/2} \cdot \frac{1}{\sqrt{1-(e^{-5x})^2}} \cdot \frac{d}{dx}(e^{-5x})$

$= \frac{1}{2}(\arcsin(e^{-5x}))^{-1/2} \cdot \frac{1}{\sqrt{1-e^{-10x}}} \cdot e^{-5x} \cdot \frac{d}{dx}(-5x)$

$= \frac{1}{2}(\arcsin(e^{-5x}))^{-1/2} \cdot \frac{1}{\sqrt{1-e^{-10x}}} \cdot e^{-5x} \cdot (-5)$

(b) $y = \left(x^2+9\right)^{3\ln x}$

$$\ln(y) = \ln\left((x^2+9)^{3\ln(x)}\right)$$

$$\ln(y) = 3\ln(x) \cdot \ln(x^2+9)$$

$$\frac{d}{dx}(\ln(y)) = \frac{d}{dx}(3\ln(x) \cdot \ln(x^2+9))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \frac{d}{dx}(3\ln(x))(\ln(x^2+9)) + (3\ln(x))\frac{d}{dx}(\ln(x^2+9))$$

$$\frac{1}{y} \cdot \frac{dy}{dx} = \left(3 \cdot \frac{1}{x}\right)(\ln(x^2+9)) + 3\ln(x)\left(\frac{1}{x^2+9} \cdot 2x\right)$$

$$\frac{dy}{dx} = y\left(\frac{3\ln(x^2+9)}{x} + \frac{6x\ln(x)}{x^2+9}\right)$$

$$\frac{dy}{dx} = (x^2+9)^{3\ln(x)}\left(\frac{3\ln(x^2+9)}{x} + \frac{6x\ln(x)}{x^2+9}\right)$$

(c) $xy^3 + x^5y = 42$

$$\frac{d}{dx}(xy^3 + x^5y) = \frac{d}{dx}(42)$$

$$\left[\frac{d}{dx}(x)(y^3) + (x)\frac{d}{dx}(y^3)\right] + \left[\frac{d}{dx}(x^5)(y) + (x^5)\frac{d}{dx}(y)\right] = 0$$

$$\left[(1)(y^3) + (x)\left(3y^2\frac{dy}{dx}\right)\right] + \left[(5x^4)(y) + (x^5)\left(\frac{dy}{dx}\right)\right] = 0$$

$$3xy^2\frac{dy}{dx} + x^5\frac{dy}{dx} = -y^3 - 5x^4y$$

$$(3xy^2 + x^5)\frac{dy}{dx} = -y^3 - 5x^4y$$

$$\frac{dy}{dx} = \frac{-y^3 - 5x^4y}{3xy^2 + x^5}$$