

Name

SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Determine the value of $g'(\pi/4)$ given that $g(x) = 10 \sin x + 4 \cos x + 100$.

$$\begin{aligned}
 g'(x) &= 10 \cos(x) - 4 \sin(x) \\
 g'\left(\frac{\pi}{4}\right) &= 10 \cos\left(\frac{\pi}{4}\right) - 4 \sin\left(\frac{\pi}{4}\right) \\
 &= 10\left(\frac{\sqrt{2}}{2}\right) - 4\left(\frac{\sqrt{2}}{2}\right) \\
 &= 5\sqrt{2} - 2\sqrt{2} \\
 &= 3\sqrt{2}
 \end{aligned}$$

2. (2 points) Determine the equation of the line which is tangent to the curve

$$y = 2e^x - 6 \Rightarrow y' = 2e^x$$

at its x -intercept.

To find x -int.,
set $y = 0$

$$0 = 2e^x - 6$$

$$6 = 2e^x$$

$$3 = e^x$$

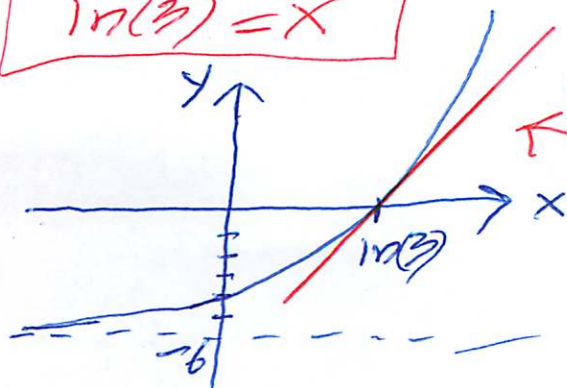
$$\ln(3) = x$$

point: $(\ln(3), 0)$

$$\begin{aligned}
 \text{slope: } y'(\ln(3)) &= 2e^{\ln(3)} \\
 &= 2 \cdot 3 \\
 &= 6
 \end{aligned}$$

$$y - 0 = 6(x - \ln(3))$$

$$y = 6x - 6\ln(3)$$



3. (2 points each) Using Leibniz notation (i.e., $\frac{dy}{dx}$, $\frac{dP}{dt}$, etc.), find derivatives for each of the following functions.

(a) $y = \left(\frac{x\sqrt{x}}{\sqrt[6]{x}}\right)^3$ (simplify your answer)

$$y = \left(\frac{x \cdot x^{1/2}}{x^{1/6}}\right)^3 = \left(\frac{x^{3/2}}{x^{1/6}}\right)^3 = \left(x^{\frac{3}{2} - \frac{1}{6}}\right)^3$$

$$= \left(x^{\frac{9}{6} - \frac{1}{6}}\right)^3 = \left(x^{\frac{8}{6}}\right)^3 = \left(x^{4/3}\right)^3 = x^4$$

$$\frac{dy}{dx} = 4x^3$$

(b) $\theta = \frac{8 \tan t}{t^5 + 6}$

$$\frac{d\theta}{dt} = \frac{\frac{d}{dt}(8 \tan(t)) \cdot (t^5 + 6) - (8 \tan(t)) \cdot \frac{d}{dt}(t^5 + 6)}{(t^5 + 6)^2}$$

$$= \frac{8 \sec^2(t) \cdot (t^5 + 6) - 8 \tan(t) \cdot 5t^4}{(t^5 + 6)^2}$$

(c) $w = 10q^8 e^q + e^{\cos(\pi/13)}$

(note: $e^{\cos(\pi/13)}$ is a constant so has deriv. 0)

$$\frac{dw}{dq} = \frac{d}{dq}(10q^8) \cdot (e^q) + (10q^8) \cdot \frac{d}{dq}(e^q) + 0$$

$$= 80q^7 e^q + 10q^8 e^q$$