Math 220 (section AD?) Quiz 3 Spring 2014

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points each) Evaluate the following limits.

(a) $\lim_{x\to 0} \frac{\sqrt{25+16x^2}-5}{2x^2}$ $= \lim_{x\to 0} \frac{(\sqrt{25+16x^2}-5)(\sqrt{25+16x^2}+5)}{2x^2(\sqrt{25+16x^2}+5)}$

$= \lim_{x\to 0} \frac{(\sqrt{25+16x^2})^2-(5)^2}{2x^2(\sqrt{25+16x^2}+5)}$

$= \lim_{x\to 0} \frac{25+16x^2-25}{2x^2(\sqrt{25+16x^2}+5)}$

$= \lim_{x\to 0} \frac{8}{\sqrt{25+16x^2}+5} = \frac{8}{10} = \frac{4}{5}$

(b) $\lim_{x\to 8^+} \frac{\ln x}{16-2x}$ $\frac{\to \ln 8}{\to 0^-}$ $= -\infty$

From graph, we see $\ln 8 > 0$

2. (2 points) Evaluate $\cos(2\arctan(3))$.

① Let $\theta = \arctan(3)$
Then $\tan(\theta) = 3$
with $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$
(actually $0 < \theta < \frac{\pi}{2}$ since $\tan\theta > 0$)

② $\tan(\theta) = 3 = \frac{3}{1}$ $\left(\frac{\text{opp}}{\text{adj}}\right)$

$1^2+3^2 = (\text{hyp})^2$
$\Rightarrow \text{hyp} = \sqrt{10}$

③ $\cos(2\arctan(3)) = \cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$= \left(\frac{1}{\sqrt{10}}\right)^2 - \left(\frac{3}{\sqrt{10}}\right)^2$

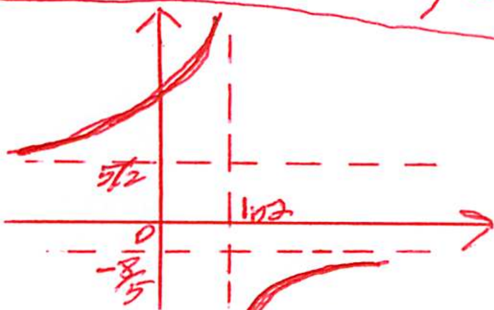
$= \frac{1}{10} - \frac{9}{10} = \frac{-8}{10} = \frac{-4}{5}$

3. (2 points) Find all horizontal asymptotes on the graph of $f(x) = \dfrac{8e^x + 25}{10 - 5e^x}$

$$\lim_{x\to\infty} \frac{8e^x+25}{10-5e^x} = \lim_{x\to\infty} \frac{8e^x+25}{10-5e^x} \cdot \frac{1/e^x}{1/e^x} = \lim_{x\to\infty} \frac{8+25/e^x}{10/e^x-5} = \frac{8+0}{0-5} = -\frac{8}{5}$$

$$\lim_{x\to-\infty} \frac{8e^x+25}{10-5e^x} = \frac{25}{10} = \frac{5}{2} \quad \left(\text{note: } \lim_{x\to-\infty} e^x = 0 \quad y=e^x\right)$$

$f(x) = \dfrac{8e^x+25}{10-5e^x}$ has horizontal asymptotes at $y = -\frac{8}{5}$ and $y = \frac{5}{2}$

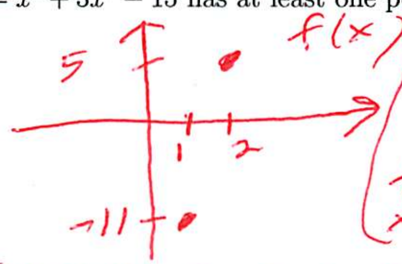


$f(x)$ also has a vertical asymptote at $x = \ln 2$

4. (2 points) Prove rigorously that $f(x) = x^3 + 3x^2 - 15$ has at least one positive root.

$f(1) = -11 < 0$

$f(2) = 5 > 0$



(we must explain why $f(x)$ crosses the x-axis on $(1,2)$)

Polynomials are continuous everywhere. Thus $f(x)$ is continuous on $[1,2]$. By the Intermediate Value Theorem, there is a c in $(1,2)$ such that $f(c) = 0$. Thus $f(x)$ has a positive root c in $(1,2)$.