

Name

Solutions

• You have 15 minutes

• No calculators

• Show sufficient work

1. (4 points) Given that $f(x) = \frac{5 + 2e^x}{4 - 3e^x}$, find a formula for $f^{-1}(x)$.

$$y = \frac{5 + 2e^x}{4 - 3e^x}$$

$$x = \frac{5 + 2e^y}{4 - 3e^y} \quad (\text{switch } x \text{ \& } y)$$

$$x(4 - 3e^y) = 5 + 2e^y \quad (\text{solve for } y)$$

$$4x - 3xe^y = 5 + 2e^y$$

$$4x - 5 = 3xe^y + 2e^y$$

$$4x - 5 = e^y(3x + 2)$$

$$\frac{4x - 5}{3x + 2} = e^y$$

$$\ln\left(\frac{4x - 5}{3x + 2}\right) = \ln(e^y)$$

$$\ln\left(\frac{4x - 5}{3x + 2}\right) = y$$

$$f^{-1}(x) = \ln\left(\frac{4x - 5}{3x + 2}\right)$$

2. (3 points) Determine all values of x which satisfy the equation below.

$$13 - 7x = 10e^{2\ln(1-x)}$$

$$13 - 7x = 10e^{\ln((1-x)^2)}$$

$$13 - 7x = 10(1-x)^2$$

$$13 - 7x = 10(1 - 2x + x^2)$$

$$13 - 7x = 10 - 20x + 10x^2$$

$$0 = 10x^2 - 13x - 3$$

$$0 = (5x+1)(2x-3)$$

$$5x+1=0 \text{ or } 2x-3=0$$

$$x = -1/5 \text{ or } x = 3/2 \text{ but } 3/2 \text{ is not in}$$

the domain of $10e^{2\ln(1-x)}$

since we only take \ln of pos. numbers.

3. (3 points) Determine a formula for an exponential function given that its graph goes through the points $(-3, 100)$, $(0, 20)$ and $(3, 4)$.

$$y = C \cdot a^x$$

$$(0, 20) \text{ on graph} \Rightarrow 20 = C \cdot a^0 \Rightarrow C = 20$$

$$y = 20 \cdot a^x$$

$$(3, 4) \text{ on graph} \Rightarrow 4 = 20 \cdot a^3 \Rightarrow a^3 = \frac{4}{20} = \frac{1}{5}$$

$$\Rightarrow a = \sqrt[3]{\frac{1}{5}} = \left(\frac{1}{5}\right)^{1/3}$$

$$y = 20 \cdot \left(\left(\frac{1}{5}\right)^{1/3}\right)^x = 20 \left(\frac{1}{5}\right)^{x/3}$$

~~it can also be written as~~
 I did not use the point $(-3, 100)$ but one can check that it satisfies our formula.