Math 220 **Quiz 12 (take-home)** **Spring 2014**

Name Solutions

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Darlayne Addabbo
- ▷ **AD2**, TR 1:00-2:50, Ben Fulan
- ▷ **ADA**, TR 8:00-8:50, Chris Bailey
- ▷ **ADB**, TR 9:00-9:50, Chris Bailey
- ▷ **ADC**, TR 10:00-10:50, Andrew McConvey
- ▷ **ADD**, TR 11:00-11:50, Diaa Taha
- ▷ **ADE**, TR 12:00-12:50, Paul Spiegelhalter
- ▷ **ADF**, TR 1:00-1:50, Diaa Taha
- ▷ **ADG**, TR 2:00-2:50, Paulina Koutsaki
- ▷ **ADH**, TR 3:00-3:50, Paulina Koutsaki
- ▷ **ADJ**, TR 9:00-9:50, Jed Chou
- ▷ **ADK**, TR 10:00-10:50, Jed Chou
- ▷ **ADL**, TR 11:00-11:50, Andrew McConvey
- ▷ **ADM**, TR 12:00-12:50, Benjamin Wright
- ▷ **ADN**, TR 1:00-1:50, Benjamin Wright
- ▷ **ADO**, TR 2:00-2:50, Paul Spiegelhalter
- ▷ **ADP**, TR 3:00-3:50, Wan-Yu Wu
- ▷ **ADQ**, TR 4:00-4:50, Wan-Yu Wu

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes, the textbook, or information found on my course home page.
- The only computational technology you may use is the basic arithmetic features of a calculator for problem 3.
- You are not allowed to search the Internet, use Wolfram Alpha, or use technology for anything beyond what is stated above.
- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, April 25th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (3 points) Use the techniques of linear approximation found in section 3.10 to obtain a good estimate for $(122)^{2/3}$ without the use of technology. Simplify and write your answer in decimal form.

$f(x) = x^{2/3}$, $f(125) = 125^{2/3} = (\sqrt[3]{125})^2 = 5^2 = 25$

$f'(x) = \frac{2}{3}x^{-1/3}$ $f'(125) = \frac{2}{3}(125)^{-1/3} = \frac{2}{3\sqrt[3]{125}} = \frac{2}{3\cdot 5} = \frac{2}{15}$

we will find the line tangent to $f(x) = x^{2/3}$ at $x = 125$.

point: $(125, f(125)) = (125, 25)$

slope: $f'(125) = 2/15$

$y - 25 = \frac{2}{15}(x-125) \Rightarrow y = 25 + \frac{2}{15}(x-125)$

$x^{2/3} \approx 25 + \frac{2}{15}(x-125)$ for x near 125

$122^{2/3} \approx 25 + \frac{2}{15}(122-125)$

$= 25 - \frac{6}{15} = 25 - \frac{2}{5} = 25 - 0.4 = 24.6$

$122^{2/3} \approx 24.6$

2. (3 points) Suppose that the polynomial $g(x)$ is an odd function and satisfies the following conditions.

- $g(3) = 7$
- $g'(3) = 4$
- $g''(3) = 3$
- $g'''(3) = 2$

Use the techniques of linear approximation found in section 3.10 to estimate the value of $g(-3.2)$. Simplify and write your answer in decimal form.

Point: $(3, g(3)) = (3, 7)$

slope: $g'(3) = 4$

tangent line at $x=3$: $y - 7 = 4(x-3) \Rightarrow y = 7 + 4(x-3)$

$g(x) \approx 7 + 4(x-3)$ for x near 3

$g(-3.2) = -g(3.2)$ since g is odd

$\approx -(7 + 4(3.2-3)) = -7.8$

3. (4 points) There is one value of x for which the line tangent to the graph of $f(x) = x^4 + x^2$ is perpendicular to the line $y = \frac{1}{20}x - 10$. Determine this x-value using Newton's Method with an initial estimate of $x_1 = -1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Your final answer should include 5 places after the decimal point.

- The line $y = \frac{1}{20}x - 10$ has slope $\frac{1}{20}$
- Each line perpendicular to this line has slope $\frac{-1}{1/20} = -20$
- Each line tangent to $f(x) = x^4 + x^2$ at x has slope $f'(x) = 4x^3 + 2x$

Thus $4x^3 + 2x = -20$

or $4x^3 + 2x + 20 = 0$

Let $g(x) = 4x^3 + 2x + 20$ and apply Newton's method to estimate a root of $g(x)$

$g'(x) = 12x^2 + 2$

$$x_{n+1} = x_n - \frac{g(x_n)}{g'(x_n)}$$

$x_1 = -1$

$$x_2 = x_1 - \frac{g(x_1)}{g'(x_1)} = -1 - \frac{g(-1)}{g'(-1)} = -1 - \frac{14}{14} = -2$$

$$x_3 = x_2 - \frac{g(x_2)}{g'(x_2)} = -2 - \frac{g(-2)}{g'(-2)} = -2 - \frac{-16}{50} = \frac{-42}{25} = -1.68$$

$$x_4 = x_3 - \frac{g(x_3)}{g'(x_3)} = -1.68 - \frac{g(-1.68)}{g'(-1.68)}$$

$$= -1.68 - \frac{-2.326528}{35.8688}$$

$$\approx -1.615137836$$

$$\approx -1.61514$$