Math 220 (section AD?) Quiz 11 Spring 2014

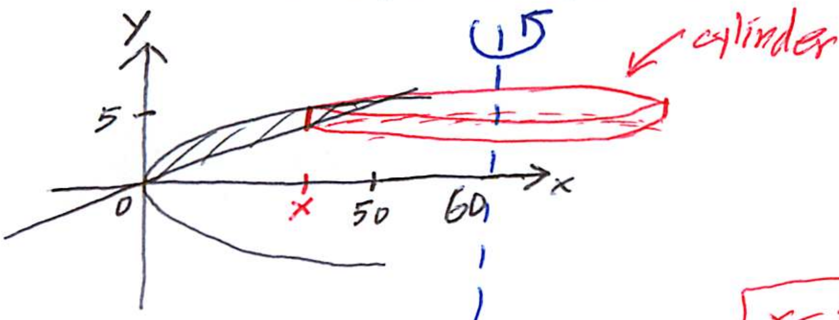
Name solutions

- You have 15 minutes
- No calculators
- Show sufficient work

1. (6 points) Let $\mathbf{R}$ be the finite region bounded by the graphs of $x = 2y^2$ and $x = 10y$. Set up, <u>but do not evaluate</u>, definite integrals which represent the volumes of the following solids.

   (a) The volume of the solid formed when $\mathbf{R}$ is revolved around the vertical line $x = 60$. Determine this volume in the following two ways.

   i. Integrate with respect to x.



intersection

$2y^2 = 10y$

$2y^2 - 10y = 0$

$2y(y-5) = 0$

$y = 0 \Rightarrow x = 0$

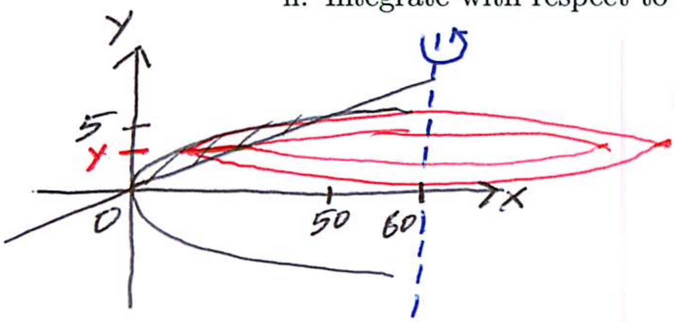
and $y = 5 \Rightarrow x = 50$

$x = 2y^2 \Rightarrow y = \pm\sqrt{\frac{x}{2}}$, $x = 10y \Rightarrow y = \frac{x}{10}$

$$V = \int_0^{50} \begin{pmatrix}\text{surface}\\ \text{area}\end{pmatrix} dx = \int_0^{50} 2\pi \cdot \text{radius} \cdot \text{height}\, dx$$

$$= \int_0^{50} 2\pi (60 - x)\left(\sqrt{\tfrac{x}{2}} - \tfrac{x}{10}\right) dx$$

   ii. Integrate with respect to y. (Use different integrands in parts *i* and *ii*.)



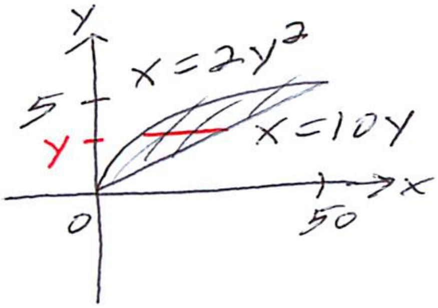
cross-section at y

Area $= \pi r_{out}^2 - \pi r_{in}^2$

$$V = \int_0^5 \begin{pmatrix}\text{cross-sectional}\\ \text{area}\end{pmatrix} dy = \int_0^5 \left(\pi r_{out}^2 - \pi r_{in}^2\right) dy$$

$$= \int_0^5 \left(\pi (60 - 2y^2)^2 - \pi (60 - 10y)^2\right) dy$$

(b) The volume of the solid with base **R** for which the cross-sections perpendicular to the y-axis are semi-circles.



Cross-section at y

$\text{area} = \frac{1}{2}\pi r^2$

$$V = \int_0^5 \left(\begin{matrix}\text{cross-sectional}\\ \text{area}\end{matrix}\right) dy = \int_0^5 \frac{1}{2}\pi (\text{radius})^2 dy$$

$$= \int_0^5 \frac{1}{2}\pi \left(\frac{\text{diameter}}{2}\right)^2 dy$$

$$= \boxed{\int_0^5 \frac{1}{2}\pi \left(\frac{10y - 2y^2}{2}\right)^2 dy}$$

2. (4 points) Find the average value of the function $f(x) = \dfrac{24x}{x^2+1}$ on the interval $[1, 7]$.

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$= \frac{1}{7-1}\int_1^7 \frac{24x}{x^2+1}\,dx$$

$$= \frac{1}{6}\int_2^{50} \frac{24}{u}\cdot\frac{1}{2}\,du$$

$$= 2\int_2^{50} \frac{1}{u}\,du$$

$$= 2\Big[\ln|u|\Big]_2^{50}$$

$$= 2\left[\ln(50) - \ln(2)\right]$$

$$= 2\ln\left(\frac{50}{2}\right)$$

$$= 2\ln(25) = 2\ln(5^2) = \boxed{4\ln(5)}$$

substitution

$u = x^2 + 1$

$\frac{du}{dx} = 2x \Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

$u(1) = 1^2 + 1 = 2$

$u(7) = 7^2 + 1 = 50$