Math 220 (section AD?) Quiz 10 Spring 2014

Name solutions

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Precisely state *The Mean Value Theorem.*

suppose the following:

① f is continuous on [a,b]

② f is differentiable on (a,b)

Then $f'(c) = \frac{f(b)-f(a)}{b-a}$ for some c in (a,b)

2. (2 points) Evaluate the definite integral. Simplify your answer.

$$\int_0^{\pi/2} \frac{\cos x}{e^{\sin x}}\,dx = \int_0^{\pi/2} e^{-\sin x}\cos x\,dx = \int_0^{-1} e^u(-du)$$

$u = -\sin x$

$\frac{du}{dx} = -\cos x$

$du = -\cos x\,dx$

$-du = \cos x\,dx$

$x = 0 \Rightarrow u = -\sin(0) = 0$

$x = \frac{\pi}{2} \Rightarrow u = -\sin(\frac{\pi}{2}) = -1$

$$= \left[-e^u\right]_0^{-1}$$

$$= -e^{-1} - (-e^0)$$

$$= -\frac{1}{e} + 1$$ ☆

3. (2 points) Suppose f is continuous everywhere and $\int_1^4 f(x)\,dx = 5$.

What is the value of $\int_0^{\sqrt{3}} 4xf\left(x^2+1\right)\,dx$?

$$\int_0^{\sqrt{3}} 4xf(x^2+1)\,dx = \int_1^4 2f(u)\,du$$

$u = x^2+1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$2du = 4x\,dx$

$x = 0 \Rightarrow u = (0)^2+1 = 1$

$x = \sqrt{3} \Rightarrow u = (\sqrt{3})^2+1 = 4$

$$= 2\int_1^4 f(u)\,du$$

$$= 2\cdot 5$$

$$= 10$$ ☆

4. (2 points) Evaluate the following indefinite integral. Hint: Use polynomial long division or some other approach to simplify the integrand.

$$\int \frac{x^5+x^3+1}{x^2+1}\,dx = \int \frac{x^3(x^2+1)+1}{x^2+1}\,dx = \int\left(\frac{x^3(x^2+1)}{x^2+1} + \frac{1}{x^2+1}\right)dx$$

$$= \int\left(x^3 + \frac{1}{x^2+1}\right)dx$$

$$= \frac{1}{4}x^4 + \arctan(x) + C$$

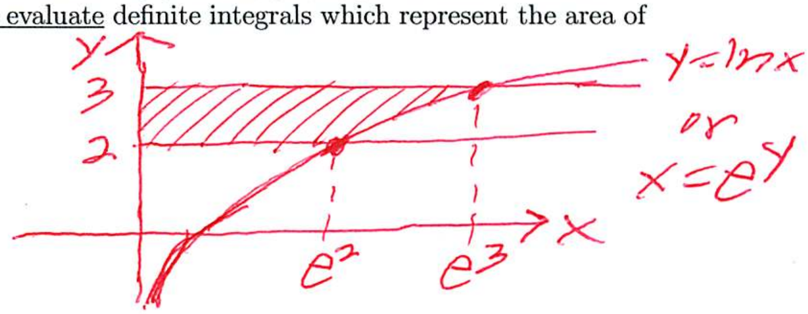
another way to simplify the integrand using polynomial long division

$$\begin{array}{r} x^3 \\ x^2+1\,\overline{)\,x^5+x^3+1} \\ \underline{x^5+x^3} \\ 1 \end{array} \Rightarrow \frac{x^5+x^3+1}{x^2+1} = x^3 + \frac{1}{x^2+1}$$

5. (2 points) Let **R** be the finite region bounded by $y = \ln x$, $y = 2$, $y = 3$ and $x = 0$. In the following manner, set up but do not evaluate definite integrals which represent the area of the region **R**.

$\ln x = 2 \Rightarrow x = e^2$

$\ln x = 3 \Rightarrow x = e^3$



(a) Integrate with respect to x.

$$\text{area} = \int_a^b (y_{top} - y_{bottom})\,dx$$

$$\text{area} = \int_0^{e^2}(3-2)\,dx + \int_{e^2}^{e^3}(3-\ln x)\,dx$$

(b) Integrate with respect to y. (The integrands in parts (a) and (b) should be different.)

$$\text{area} = \int_c^d (x_{right} - x_{left})\,dy$$

$$\text{area} = \int_2^3 (e^y - 0)\,dy$$