1. (2 points) Precisely state The Mean Value Theorem.

Suppose the following:
1. 
2. 
Then \( f'(c) = \frac{f(b) - f(a)}{b - a} \) for some \( c \) in \( (a, b) \).

2. (2 points) Evaluate the definite integral. Simplify your answer.

\[
\int_0^{\pi/2} \frac{\cos x}{e^{\sin x}} \, dx = \int_0^{\pi/2} e^{-\sin x} \cos x \, dx = \int_0^1 e^u (-du)
\]

\( u = -\sin x \)
\( du = -\cos x \, dx \)
\( x = 0 \Rightarrow u = -\sin(0) = 0 \)
\( x = \frac{\pi}{2} \Rightarrow u = -\sin(\frac{\pi}{2}) = -1 \)

\( = e^{-1} (e^0) = \frac{1}{e} + 1 \)

3. (2 points) Suppose \( f \) is continuous everywhere and \( \int_1^4 f(x) \, dx = 5 \).

What is the value of \( \int_0^{\sqrt{3}} 4x f(x^2 + 1) \, dx \)?

\[
\int_0^{\sqrt{3}} 4x f(x^2 + 1) \, dx = \int_1^4 2f(u) \, du
\]

\( u = x^2 + 1 \)
\( du = 2x \, dx \)
\( x = 0 \Rightarrow u = (0)^2 + 1 = 1 \)
\( x = \sqrt{3} \Rightarrow u = (\sqrt{3})^2 + 1 = 4 \)

\( = 2 \int_1^4 2f(u) \, du \)
\( = 2 \cdot 5 \)
\( = 10 \)

\( \star \)
4. (2 points) Evaluate the following indefinite integral. Hint: Use polynomial long division or some other approach to simplify the integrand.

\[
\int \frac{x^5 + x^3 + 1}{x^2 + 1} \, dx = \int \frac{x^3(x^2 + 1) + 1}{x^2 + 1} \, dx = \int \left( \frac{x^3(x^2 + 1)}{x^2 + 1} + \frac{1}{x^2 + 1} \right) \, dx = \int \left( x^3 + \frac{1}{x^2 + 1} \right) \, dx = \frac{1}{4} x^4 + \arctan(x) + C
\]

Another way to simplify the integrand:

Using polynomial long division:

\[
\frac{x^5 + x^3 + 1}{x^2 + 1} = x^3 + \frac{1}{x^2 + 1}
\]

5. (2 points) Let \( R \) be the finite region bounded by \( y = \ln x, y = 2, y = 3 \) and \( x = 0 \). In the following manner, set up but do not evaluate definite integrals which represent the area of the region \( R \).

\[
\begin{align*}
\ln x = 2 & \Rightarrow x = e^2 \\
\ln x = 3 & \Rightarrow x = e^3
\end{align*}
\]

(a) Integrate with respect to \( x \).

\[
\text{area} = \int_{e^2}^{e^3} (\text{top} - \text{bottom}) \, dx = \int_{e^2}^{e^3} (3 - \ln x) \, dx + \int_{e^2}^{e^3} (3 - 2) \, dx
\]

(b) Integrate with respect to \( y \). (The integrands in parts (a) and (b) should be different.)

\[
\text{area} = \int_c^d (x_{\text{right}} - x_{\text{left}}) \, dy
\]

\[
\text{area} = \int_2^3 (e^y - 0) \, dy
\]