



1. (6 points) Circle the domain of the function  $f(x) = \frac{\sqrt{9-x^2} + \ln(x+1) + e^{x-2}}{\sqrt[3]{x-4}}$

(a)  $(-3, -1]$

(b)  $(-3, 2]$

(c)  $(-3, 3]$

(d)  $(-3, 4]$

(e)  $(-1, 2]$

(f)  $(-1, 3]$

(g)  $(-1, 4]$

(h)  $(2, 3]$

(i)  $(2, 4]$

(j)  $(3, 4]$

(k)  $(-\infty, \infty)$

2. (6 points) For the function  $g$ , the following finite limit exists.

$$\lim_{\alpha \rightarrow 0} \frac{g(b+\alpha) - g(b)}{\alpha}$$

From the choices below, circle the three statements which must be true.

(a)  $g$  is one-to-one.

(b)  $g$  is an odd function.

(c)  $g$  is an even function.

(d)  $g$  is differentiable at 0.

(e)  $g$  is differentiable at  $b$ .

(f)  $g$  is differentiable at  $\alpha$ .

(g)  $g$  is continuous at 0.

(h)  $g$  is continuous at  $b$ .

(i)  $g$  is continuous at  $\alpha$ .

(j)  $\lim_{x \rightarrow 0} g(x) = g(0)$

(k)  $\lim_{x \rightarrow b} g(x) = g(b)$

(l)  $\lim_{x \rightarrow \alpha} g(x) = g(\alpha)$

3. (10 points) Let  $f(x) = 5x^2 - 4x$ .

Use the definition of a derivative as a limit to prove that  $f'(x) = 10x - 4$ .

Show each step in your calculation and be sure to use proper terminology in each step of your proof.

4. (6 points each) Evaluate the following quantities and simplify your answer.

(a)  $\tan(\arcsin(4/5))$

(b)  $e^{2\ln(5)} + 6\ln(\sqrt{e})$

5. (10 points) The function  $f(x) = 12 - 3x^2$  with restricted domain  $x \leq 0$  is one-to-one. Determine a formula for its inverse  $f^{-1}(x)$ .

6. (12 points) Find a formula for an exponential function  $f(x)$  for which  $f(0) = 3$  and  $f(2) = 12$ . For which real number  $b$  does  $f(b) = 500$  ?

7. (8 points) For a given acute angle  $\theta$ , it is known that  $\cos \theta = 4/5$ . Determine the value of  $\cos(\pi - 2\theta)$ . Write your answer as a simplified fraction.

8. (7 points each) Evaluate the following limits without the use of derivatives. Show sufficient justification for each answer. An answer of 'does not exist' is not sufficient. For infinite limits you must state if it is  $\infty$  or  $-\infty$ .

(a)  $\lim_{x \rightarrow \infty} \frac{\sin x}{x}$

(b)  $\lim_{x \rightarrow \pi^+} \frac{\cos(\pi - x)}{\pi - x}$

(c)  $\lim_{x \rightarrow -\infty} \frac{2x^3 + 1}{5x + 3x^3}$

(d)  $\lim_{x \rightarrow 5} \left( \frac{10}{x^2 - 25} - \frac{1}{x - 5} \right)$

9. (8 points) Does the function  $f(x) = \frac{1 - e^x}{1 - e^{2x}}$  have a vertical asymptote at  $x = 0$  ? Explain your reasoning.

**Students – do not write on this page!**

1. (6 points) \_\_\_\_\_

2. (6 points) \_\_\_\_\_

3. (10 points) \_\_\_\_\_

4. (12 points) \_\_\_\_\_

5. (10 points) \_\_\_\_\_

6. (12 points) \_\_\_\_\_

7. (8 points) \_\_\_\_\_

8a. (7 points) \_\_\_\_\_

8b. (7 points) \_\_\_\_\_

8c. (7 points) \_\_\_\_\_

8d. (7 points) \_\_\_\_\_

9. (8 points) \_\_\_\_\_

**TOTAL (100 points)** \_\_\_\_\_