Math 220 (section AD?) **Quiz 9** **Spring 2013**

Name SOLUTIONS

- You have 20 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

$$\lim_{n\to\infty}\sum_{k=1}^{n}\left(\frac{5}{n}+\frac{20k}{n^2}+\frac{100}{n^2}\right)$$

$$=\lim_{n\to\infty}\left(\sum_{k=1}^{n}\frac{5}{n}+\sum_{k=1}^{n}\frac{20k}{n^2}+\sum_{k=1}^{n}\frac{100}{n^2}\right)$$

$$=\lim_{n\to\infty}\left(\frac{5}{n}\cdot n+\frac{20}{n^2}\sum_{k=1}^{n}k+\frac{100}{n^2}\cdot n\right)$$

$$=\lim_{n\to\infty}\left(5+\frac{20}{n^2}\cdot\frac{n(n+1)}{2}+\frac{100}{n}\right)$$

$$=5+\frac{20}{2}+0$$

$$=15$$

2. (2 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be n and k. You do not need to evaluate this limit.

$$\int_3^8 \frac{\sin x}{e^{2x}}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n}\left[\ \frac{\sin\left(3+k\cdot\frac{5}{n}\right)}{e^{2\left(3+k\cdot\frac{5}{n}\right)}}\cdot\frac{5}{n}\ \right]$$

$$\int_3^8 \frac{\sin x}{e^{2x}}\,dx = \lim_{n\to\infty}\sum_{k=1}^{n} f(x_k^*)\Delta x$$

$f(x) = \frac{\sin x}{e^{2x}}$

$\Delta x = \frac{8-3}{n} = \frac{5}{n}$

$x_k^* = 3 + k\cdot\Delta x$
$= 3 + k\cdot\frac{5}{n}$

note: I used a right Riemann sum.
For a left Riemann sum, change each k to k-1.
For a midpoint Riemann sum, change each k to k-0.5.

3. (2 points) Suppose f is a polynomial for which $\int_1^7 f(x)\,dx = 12$ and $\int_5^7 f(x)\,dx = 4$. What is the value of $\int_5^1 f(x)\,dx$?

$$\int_5^1 f(x)\,dx = -\int_1^5 f(x)\,dx$$
$$= -\left(\int_1^7 f(x)\,dx + \int_7^5 f(x)\,dx\right)$$
$$= -\left(\int_1^7 f(x)\,dx - \int_5^7 f(x)\,dx\right)$$
$$= -(12 - 4)$$
$$= -8$$

4. (2 points) Evaluate the following indefinite integral.

$$\int \frac{(\sin x - \cos x)^2 + \sin(2x)}{\cos^2 x}\,dx$$

$$= \int \frac{(\sin^2 x - 2\sin x\cos x + \cos^2 x) + 2\sin x \cos x}{\cos^2 x}\,dx$$

$$= \int \frac{\sin^2 x + \cos^2 x}{\cos^2 x}\,dx$$

$$= \int \frac{1}{\cos^2 x}\,dx$$

$$= \int \sec^2 x\,dx$$

$$= \tan x + C$$

5. (2 points) Evaluate and simplify the following definite integral.

$$\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2}\,dx$$

$$= \frac{1}{3/2} x^{3/2} \Big|_1^4$$

$$= \frac{2}{3} x^{3/2} \Big|_1^4$$

$$= \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$$

$$= \frac{2}{3} \cdot 8 - \frac{2}{3}$$

$$= 14/3$$