

Name

SOLUTIONS

• You have 15 minutes

• No calculators

• Show sufficient work

1. (4 points) The acceleration due to gravity near the surface of some planet is -8 m/s^2 . An object is shot upward from the surface of this planet and 12 seconds later it has fallen back to the surface. What is the velocity of this object 2 seconds after being shot?

(acc) $s''(t) = -8$

(vel) $s'(t) = -8t + C$

(pos) $s(t) = -4t^2 + Ct + D$

$$s(0) = 0 \Rightarrow -4(0)^2 + C(0) + D = 0 \Rightarrow D = 0$$

$$s(t) = -4t^2 + Ct$$

$$s(12) = 0 \Rightarrow -4(12)^2 + C(12) = 0 \Rightarrow C = \frac{4(12)^2}{12} = 48$$

$$s(t) = -4t^2 + 48t$$

and $s'(t) = -8t + 48$

$$s'(2) = -8(2) + 48 = 32 \text{ m/s}$$

alternate
solution

object attains its max. height
at $t = 6$ sec, thus $s'(6) = 0 \Rightarrow$

$$-8(b) + c = 0 \Rightarrow c = 48$$

thus, $s'(t) = -8t + 48$ and

$$s'(2) = 32 \text{ m/s}$$

2. (4 points) Find a formula for $f(x)$ given that $f''(x) = \frac{3 + 10\sqrt{x}}{\sqrt{x}}$, $f(1) = 14$ and $f'(1) = 18$.

$$f''(x) = \frac{3}{\sqrt{x}} + \frac{10\sqrt{x}}{\sqrt{x}} = 3x^{-1/2} + 10$$

$$f'(x) = 3 \cdot \frac{1}{1/2} x^{1/2} + 10x + C = 6x^{1/2} + 10x + C$$

$$f'(1) = 18 \Rightarrow 6(1)^{1/2} + 10(1) + C = 18 \Rightarrow C = 2$$

$$f'(x) = 6x^{1/2} + 10x + 2$$

$$f(x) = 6 \cdot \frac{1}{3/2} x^{3/2} + 10 \cdot \frac{1}{2} x^2 + 2x + D$$

$$f(x) = 4x^{3/2} + 5x^2 + 2x + D$$

$$f(1) = 14 \Rightarrow 4(1)^{3/2} + 5(1)^2 + 2(1) + D = 14 \Rightarrow D = 3$$

$$f(x) = 4x^{3/2} + 5x^2 + 2x + 3$$

3. (2 points) When Carl rode his bicycle yesterday, his speeds were always increasing between noon and 1:00 PM. His speeds are recorded every 15 minutes and listed in the table below.

Time	12:00	12:15	12:30	12:45	1:00
Speed (miles/hour)	12.5	15.0	20.0	22.0	25.0

$$15 \text{ min} = \frac{1}{4} \text{ hr}$$

- (a) Approximate the total distance that Carl traveled between noon and 1:00 PM by using a left Riemann sum with $\Delta t = 15$ minutes.

$$\text{distance} \approx (12.5 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) + (15.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) \\ + (20.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) + (22.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr})$$

$$\text{distance} \approx 17.375 \text{ mi}$$

- (b) Approximate the total distance that Carl traveled between noon and 1:00 PM by using a right Riemann sum with $\Delta t = 15$ minutes.

$$\text{distance} \approx (15.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) + (20.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) \\ + (22.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr}) + (25.0 \frac{\text{mi}}{\text{hr}}) (\frac{1}{4} \text{ hr})$$

$$\text{distance} \approx 20.5 \text{ mi}$$