Math 220 **Quiz 7 (take-home)** **Spring 2013**

Name SOLUTIONS

(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Nick Andersen
- ▷ **AD2**, TR 1:00-2:50, Sarah Loeb
- ▷ **ADA**, TR 8:00-8:50, Lisa Hickok
- ▷ **ADB**, TR 9:00-9:50, Sneha Chaubey
- ▷ **ADC**, TR 10:00-10:50, Sneha Chaubey
- ▷ **ADD**, TR 11:00-11:50, Tom Mahoney
- ▷ **ADE**, TR 12:00-12:50, Tom Mahoney
- ▷ **ADF**, TR 1:00-1:50, Lisa Hickok
- ▷ **ADG**, TR 2:00-2:50, Nathan Rehfuss
- ▷ **ADH**, TR 3:00-3:50, Nathan Rehfuss
- ▷ **ADJ**, TR 9:00-9:50, Dan Schultz
- ▷ **ADK**, TR 10:00-10:50, Dan Schultz
- ▷ **ADL**, TR 11:00-11:50, Derrek Yager
- ▷ **ADM**, TR 12:00-12:50, Derrek Yager
- ▷ **ADN**, TR 1:00-1:50, Ben Fulan
- ▷ **ADO**, TR 2:00-2:50, Ben Fulan
- ▷ **ADP**, TR 3:00-3:50, Mahmood Etedadi Aliabadi
- ▷ **ADQ**, TR 4:00-4:50, Mahmood Etedadi Aliabadi

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, March 8th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (3 points) Evaluate $\lim\limits_{x\to 0} \dfrac{2e^{5x}-25x^2-10x-2}{2x^3}$

$$\lim_{x\to 0} \frac{2e^{5x}-25x^2-10x-2 \quad \nearrow 0}{2x^3 \quad \to 0}$$

Ⓛ $$= \lim_{x\to 0} \frac{10e^{5x}-50x-10 \quad \nearrow 0}{6x^2 \quad \to 0}$$ by l'Hospitals Rule

Ⓛ $$= \lim_{x\to 0} \frac{50e^{5x}-50 \quad \nearrow 0}{12x \quad \to 0}$$ by l'Hospitals Rule

Ⓛ $$= \lim_{x\to 0} \frac{250e^{5x} \nearrow 250}{12 \quad \to 12}$$ by l'Hospital's Rule

$$= \frac{250}{12}$$

$$= \boxed{\frac{125}{6}}$$

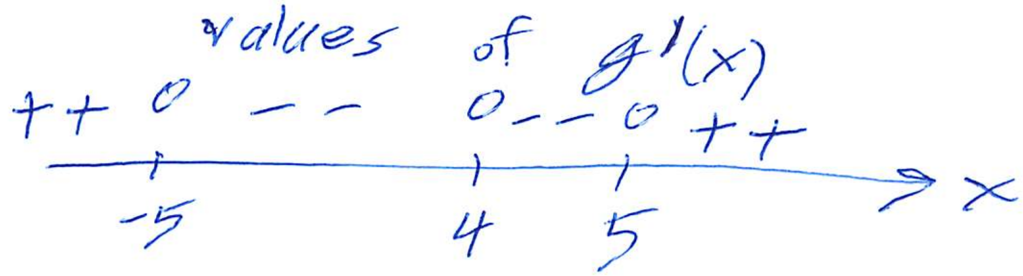
2. (4 points) Suppose that $g(x)$ is a function whose first derivative is given below.

$$g'(x) = \frac{8e^x\left(x^2+36\right)\left(x^2-25\right)(x-4)^{10}}{\ln\left(x^2+9\right)}$$

Find all critical numbers of $g(x)$. At each critical number, state whether $g(x)$ has a local maximum, local minimum or neither at that point.

since $8e^x > 0$, $x^2+36>0$, and $\ln(x^2+9)>0$, the only critical numbers for $g(x)$ are at $x^2-25=0$ or $(x-4)^{10}=0$

Critical numbers: $x=-5$, $x=5$, $x=4$



At $x=-5$ g has a local max

At $x=4$ g has neither a local max nor a local min

At $x=5$ g has a local min

(we used the first derivative test)

3. (3 points) Suppose M is the largest slope on the graph of $f(x) = \frac{3}{x^2+6}$. What is the value of M?

$$f'(x) = \frac{(3)'(x^2+6) - (3)(x^2+6)'}{(x^2+6)^2} = \frac{-6x}{(x^2+6)^2}$$

values of f'

$+++ \quad 0 \quad ---$ x

0

$\Rightarrow$ f incr. on $(-\infty, 0]$
and f decr. on $[0, \infty)$

$$f''(x) = \frac{(-6x)'(x^2+6)^2 - (-6x)((x^2+6)^2)'}{(x^2+6)^4} = \frac{-6(x^2+6)^2 + 6x(2(x^2+6)(2x))}{(x^2+6)^4}$$

$$= \frac{(x^2+6)\left[-6(x^2+6) + 24x^2\right]}{(x^2+6)^4} = \frac{18x^2-36}{(x^2+6)^3} = \frac{18(x^2-2)}{(x^2+6)^3}$$

values of f''

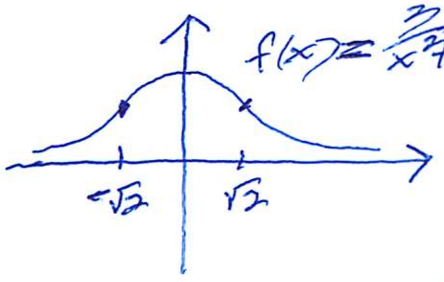
$++ \quad 0 \quad --- \quad 0 \quad ++$

$-\sqrt{2} \qquad \sqrt{2}$

$\Rightarrow$ f conc. up on $(-\infty, -\sqrt{2})$ and $(\sqrt{2}, \infty)$
and f conc. down on $(-\sqrt{2}, \sqrt{2})$

noting that $f(x) = \frac{3}{x^2+6} > 0$ for all x

and $\lim_{x\to\infty} f(x) = \lim_{x\to-\infty} f(x) = 0$ we obtain the graph.



we see the largest slope occurs at the inflection point $(-\sqrt{2}, f(-\sqrt{2})) = (-\sqrt{2}, \frac{3}{8})$.

The slope here is

$$f'(-\sqrt{2}) = \frac{-6(-\sqrt{2})}{((-\sqrt{2})^2+6)^2} = \frac{6\sqrt{2}}{64} = \boxed{\frac{3\sqrt{2}}{32}}$$

Another approach is to see that maximizing the slope amounts to maximizing the quantity $f'(x)$. Thus we will set its derivative $f''(x)$ equal to 0 and determine the maximum value of $f'(x)$.