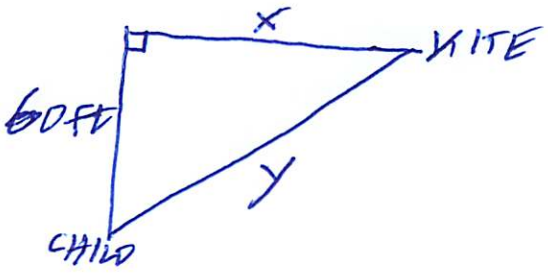
Math 220 (section AD?) **Quiz 6** **Spring 2013**

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (3 points) A child is flying a kite and is letting out 2.4 feet of string per second. If the kite stays 60 feet above the ground as it moves horizontally, then at what speed is the kite moving when there is 100 feet of string let out? You should assume that there is no sag in the string.



GIVEN $\frac{dy}{dt} = 2.4 \text{ ft/s}$

WANT $\left.\frac{dx}{dt}\right|_{y=100 \text{ ft}}$

$x^2 + 60^2 = y^2$

$\frac{d}{dt}(x^2 + 60^2) = \frac{d}{dt}(y^2)$

$2x\frac{dx}{dt} + 0 = 2y\frac{dy}{dt}$

$\boxed{x\frac{dx}{dt} = y\frac{dy}{dt}}$

Note that at $y = 100$, we get

$x^2 + 60^2 = 100^2$

$x = \sqrt{100^2 - 60^2}$

$= \sqrt{10000 - 3600}$

$= \sqrt{6400}$

$= 80$

$80 \cdot \frac{dx}{dt} = 100 \cdot (2.4)$

$\frac{dx}{dt} = \frac{240}{80}$

$\frac{dx}{dt} = 3 \text{ ft/s}$

2. (3 points) A ball is thrown straight up from an initial height of 40 feet above the ground. Until the ball hits the ground, the function $h = -16t^2+24t+40$ represents the ball's height in feet above ground level t seconds after it was thrown. What is the velocity of the ball at the instant it hits the ground?

Set $h=0$ to find when the ball hits the ground.

$-16t^2+24t+40=0$

$-8(2t^2-3t-5)=0$

$-8(t+1)(2t-5)=0 \Rightarrow t=-1$ or $t=5/2$

velocity is $h'=-32t+24$

$h'(\frac{5}{2}) = -32(\frac{5}{2})+24$

$= -56$ ft/s

3. (4 points) Solve the following differential equations given that the graph of each solution goes through the point $(0, 3)$. You must use the given variables.

(a) $\frac{dq}{dv} = 0.5v$

$q = 0.5 \cdot \frac{1}{2}v^2 + C$

$q = 0.25v^2 + C$

$3 = 0.25(0)^2 + C \Rightarrow C = 3$

$q = 0.25v^2 + 3$

(b) $\frac{dq}{dv} = 0.5q$

$q = Ce^{0.5v}$

$3 = Ce^{0.5(0)} \Rightarrow C = 3$

$q = 3e^{0.5v}$