Math 220 (section AD?) Quiz 5 Spring 2013

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Compute $g'(x)$ given that $g(x) = \sqrt{\tan(x^6 + 3x)}$.

$$g(x) = (\tan(x^6+3x))^{1/2}$$

$$g'(x) = \frac{1}{2}(\tan(x^6+3x))^{-1/2} \cdot (\tan(x^6+3x))'$$

$$g'(x) = \frac{1}{2}(\tan(x^6+3x))^{-1/2} \cdot \sec^2(x^6+3x) \cdot (x^6+3x)'$$

$$g'(x) = \frac{1}{2}(\tan(x^6+3x))^{-1/2} \cdot \sec^2(x^6+3x) \cdot (6x^5+3)$$

2. (3 points) Given $h(t) = \ln\left(t^3 + 4t^2 + 6\right)$, find its second derivative $h''(t)$.

$$h'(t) = \frac{1}{t^3+4t^2+6} \cdot (t^3+4t^2+6)'$$

$$h'(t) = \frac{1}{t^3+4t^2+6} \cdot (3t^2+8t)$$

$$h'(t) = \frac{3t^2+8t}{t^3+4t^2+6}$$

$$h''(t) = \frac{(3t^2+8t)'(t^3+4t^2+6) - (3t^2+8t)(t^3+4t^2+6)'}{(t^3+4t^2+6)^2}$$

$$h''(t) = \frac{(6t+8)(t^3+4t^2+6) - (3t^2+8t)(3t^2+8t)}{(t^3+4t^2+6)^2}$$

3. (3 points) Find the equation of the line tangent to the curve $x^3 + y^3 = xy^2 + 5$ at the point $(1, 2)$.

Note that $(1,2)$ is on the curve since $1^3+2^3=1\cdot 2^2+5$.

$\frac{d}{dx}(x^3+y^3)=\frac{d}{dx}(xy^2+5)$

$3x^2+3y^2\cdot\frac{dy}{dx}=\frac{d}{dx}(x)\cdot y^2+x\cdot\frac{d}{dx}(y^2)+0$

$3x^2+3y^2\frac{dy}{dx}=1\cdot y^2+x\cdot 2y\cdot\frac{dy}{dx}$

$3y^2\frac{dy}{dx}-2xy\frac{dy}{dx}=y^2-3x^2$

$\frac{dy}{dx}(3y^2-2xy)=y^2-3x^2$

$\frac{dy}{dx}=\frac{y^2-3x^2}{3y^2-2xy}$

POINT: $(x,y)=(1,2)$

SLOPE: $\left.\frac{dy}{dx}\right|_{(x,y)=(1,2)}$

$=\frac{2^2-3(1)^2}{3(2)^2-2(1)(2)}$

$=\frac{1}{8}$

TANGENT LINE:

$y-2=\frac{1}{8}(x-1)$

or

$y=\frac{1}{8}x+\frac{15}{8}$

4. (2 points) Compute $\frac{dy}{dx}$ given that $y = (\sin x)^{x^2}$.

$\ln y=\ln\left((\sin x)^{x^2}\right)$

$\ln y = x^2\cdot\ln(\sin x)$

$\frac{d}{dx}(\ln y)=\frac{d}{dx}(x^2\cdot\ln(\sin x))$

$\frac{1}{y}\cdot\frac{dy}{dx}=\frac{d}{dx}(x^2)\cdot\ln(\sin x)+x^2\cdot\frac{d}{dx}(\ln(\sin x))$

$\frac{1}{y}\cdot\frac{dy}{dx}=2x\cdot\ln(\sin x)+x^2\cdot\frac{1}{\sin x}\cdot\frac{d}{dx}(\sin x)$

$\frac{1}{y}\cdot\frac{dy}{dx}=2x\cdot\ln(\sin x)+x^2\cdot\frac{1}{\sin x}\cdot\cos x$

$\frac{dy}{dx}=y\left(2x\cdot\ln(\sin x)+\frac{x^2\cdot\cos x}{\sin x}\right)$

$\frac{dy}{dx}=(\sin x)^{x^2}\left(2x\ln(\sin x)+\frac{x^2\cos x}{\sin x}\right)$