Math 220 (section AD?) **Quiz 3** **Spring 2013**

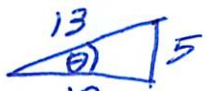
Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (2 points) Evaluate $\sin(2\arcsin(5/13))$.

Let $\theta = \arcsin(5/13)$

Then $\sin\theta = 5/13$, $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ (actually $0 \le \theta \le \frac{\pi}{2}$ since sine is pos.)



From Pythagorean Theorem

$$\sin\left(2\arcsin\left(\tfrac{5}{13}\right)\right) = \sin(2\theta) = 2\sin\theta\cos\theta = 2\left(\tfrac{5}{13}\right)\left(\tfrac{12}{13}\right) = \frac{120}{169}$$

2. (3 points) Let $f(x) = \dfrac{10e^x + 1}{6 - 2e^x}$.

(a) Find all horizontal asymptotes on the graph of $f(x)$.

$$\lim_{x\to\infty} \frac{10e^x+1}{6-2e^x} = \lim_{x\to\infty} \frac{10e^x+1}{6-2e^x}\cdot\frac{1/e^x}{1/e^x} = \lim_{x\to\infty}\frac{10+\frac{1}{e^x}}{\frac{6}{e^x}-2} = \frac{10}{-2} = -5$$

$$\lim_{x\to-\infty}\frac{10e^x+1}{6-2e^x} = \frac{1}{6}$$ since $e^x \to 0$ as $x \to -\infty$

horizontal asymptotes: $y = -5$, $y = \frac{1}{6}$

(b) Find all vertical asymptotes on the graph of $f(x)$.

denominator $= 0$ when $6 - 2e^x = 0$

$6 = 2e^x$

$3 = e^x$

$x = \ln 3$

$$\lim_{x\to\ln 3^-}\frac{\overbrace{10e^x+1}^{\to 31}}{\underbrace{6-2e^x}_{\to 0^+}} = \infty$$

$$\lim_{x\to\ln 3^+}\frac{\overbrace{10e^x+1}^{\to 31}}{\underbrace{6-2e^x}_{\to 0^-}} = -\infty$$

either limit is enough to show there is a vertical asymptote at $x = \ln 3$

3. (2 points each) Evaluate the following limits.

(a) $\lim_{x \to e^+} \frac{x^2}{1 - \ln x}$ $\to e^2$ (pos) $= -\infty$

$\to 0^-$ (neg)

(b) $\lim_{x \to 0} \frac{\sqrt{9+x} - 3}{2x}$ $= \lim_{x \to 0} \frac{\sqrt{9+x} - 3}{2x} \cdot \frac{\sqrt{9+x} + 3}{\sqrt{9+x} + 3}$

$= \lim_{x \to 0} \frac{(\sqrt{9+x})^2 - (3)^2}{2x(\sqrt{9+x} + 3)}$

$= \lim_{x \to 0} \frac{x}{2x(\sqrt{9+x} + 3)}$

$= \lim_{x \to 0} \frac{1}{2(\sqrt{9+x} + 3)}$

$= \frac{1}{2(\sqrt{9+0} + 3)} = \frac{1}{12}$

4. (1 point) Which one of the following equations must hold in order for a function w to be continuous at a number b ?

(a) $\lim_{x \to 0} w(x) = b$	(b) $\lim_{x \to 0} w(x) = 0$	(c) $\lim_{x \to 0} w(x) = w(b)$
(d) $\lim_{x \to b} w(x) = b$	(e) $\lim_{x \to b} w(x) = 0$	(f) $\lim_{x \to b} w(x) = w(b)$
(g) $\lim_{x \to \infty} w(x) = b$	(h) $\lim_{x \to \infty} w(x) = 0$	(i) $\lim_{x \to \infty} w(x) = w(b)$