Math 220 (section AD?) Quiz 2 Spring 2013

Name SOLUTIONS

- You have 15 minutes
- No calculators
- Show sufficient work

1. (4 points) A bacterial culture starts with 200 bacteria and triples in size every 2 hours.

   (a) Find a formula for the number of bacteria as a function of the number of hours since its population was 200.

t	P
0	200
2	600
4	1800
6	5400
⋮	⋮

$P = C \cdot a^t$

at $(t,P) = (0,200)$ we get

$200 = C \cdot a^0 \Rightarrow C = 200$

$P = 200 \cdot a^t$

at $(t,P) = (2,600)$ we get

$600 = 200 \cdot a^2 \Rightarrow a = \pm\sqrt{3}$

using $a = \sqrt{3}$ we obtain

$$P = 200 \cdot (\sqrt{3})^t = 200 \cdot 3^{t/2}$$

   (b) At what time is the population equal to 1000 ?

$$1000 = 200 \cdot 3^{t/2}$$

$$5 = 3^{t/2}$$

$$\ln(5) = \ln(3^{t/2})$$

$$\ln(5) = \frac{t}{2}\ln(3)$$

$$t = \frac{2\ln(5)}{\ln(3)} \text{ hours}$$

2. (2 points) Given that $g(x) = \ln\left(4 + \sqrt[3]{x}\right)$, find a formula for $g^{-1}(x)$.

$y = \ln(4+\sqrt[3]{x})$

$x = \ln(4+\sqrt[3]{y})$ (switch x & y)

$e^x = 4+\sqrt[3]{y}$

$e^x - 4 = \sqrt[3]{y}$

$y = (e^x-4)^3$ (solve for y)

$g^{-1}(x) = (e^x-4)^3$

3. (2 points) Determine all values of x which satisfy the equation below.

$$e^{3\ln(5x+7)} = 8$$

$e^{\ln((5x+7)^3)} = 8$

$(5x+7)^3 = 8$

$5x+7 = \sqrt[3]{8} = 2$

$5x = -5$

$x = -1$

4. (2 points) Find the domain of the function $f(x) = \dfrac{e^{2x}+9}{e^{2x}-100}$

the denominator equals 0 when

$e^{2x} = 100 \Rightarrow \ln(e^{2x}) = \ln(100) \Rightarrow 2x = \ln(100)$

$\Rightarrow x = \frac{1}{2}\ln(100) = \frac{1}{2}\ln(10^2) = \ln(10)$

domain of f: $(-\infty, \ln(10)) \cup (\ln(10), \infty)$