Math 220 Quiz 12 (take-home) Spring 2013

Name Solutions

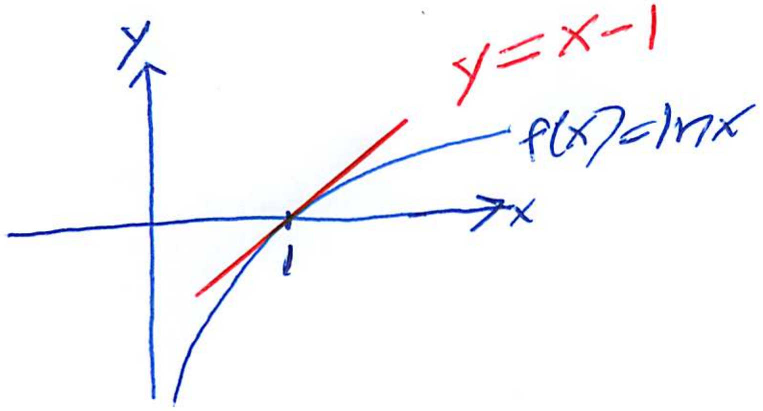
(circle your TA discussion section)

- ▷ **AD1**, TR 9:00-10:50, Nick Andersen
- ▷ **AD2**, TR 1:00-2:50, Sarah Loeb
- ▷ **ADA**, TR 8:00-8:50, Lisa Hickok
- ▷ **ADB**, TR 9:00-9:50, Sneha Chaubey
- ▷ **ADC**, TR 10:00-10:50, Sneha Chaubey
- ▷ **ADD**, TR 11:00-11:50, Tom Mahoney
- ▷ **ADE**, TR 12:00-12:50, Tom Mahoney
- ▷ **ADF**, TR 1:00-1:50, Lisa Hickok
- ▷ **ADG**, TR 2:00-2:50, Nathan Rehfuss
- ▷ **ADH**, TR 3:00-3:50, Nathan Rehfuss
- ▷ **ADJ**, TR 9:00-9:50, Dan Schultz
- ▷ **ADK**, TR 10:00-10:50, Dan Schultz
- ▷ **ADL**, TR 11:00-11:50, Derrek Yager
- ▷ **ADM**, TR 12:00-12:50, Derrek Yager
- ▷ **ADN**, TR 1:00-1:50, Ben Fulan
- ▷ **ADO**, TR 2:00-2:50, Ben Fulan
- ▷ **ADP**, TR 3:00-3:50, Mahmood Etedadi Aliabadi
- ▷ **ADQ**, TR 4:00-4:50, Mahmood Etedadi Aliabadi

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else's work.
- You may use your notes or the textbook.
- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.
- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, April 19th.
- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.
- Be sure that the pages are nicely stapled – do not just fold the corners.
- **Note to TAs and Tutors – you should not help students with these specific problems or go over solutions until after 5pm Friday.**

1. (6 points) Use the techniques of linear approximation to estimate the following quantities. Write each answer in decimal form. No technology is allowed on this problem.

(a) $\ln(0.9)$



we find the line tangent to $f(x) = \ln x$ at $x = 1$

Point: $(1, f(1)) = (1, \ln(1)) = (1, 0)$

$f'(x) = \frac{1}{x}$

slope: $f'(1) = \frac{1}{1} = 1$

$y - 0 = 1 \cdot (x - 1)$

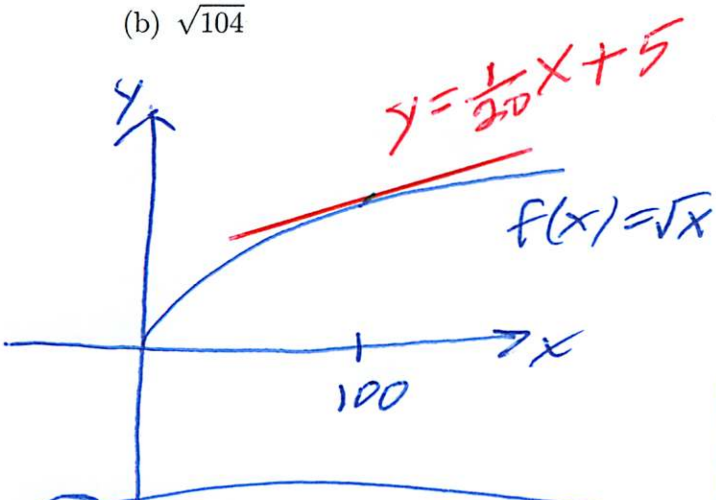
$y = x - 1$

$\ln x \approx x - 1$ for x near 1

$\ln(0.9) \approx 0.9 - 1$

$\ln(0.9) \approx -0.1$ → from graphs, the exact value of $\ln(0.9)$ is less than our estimate

(b) $\sqrt{104}$



we find the line tangent to $f(x) = \sqrt{x}$ at $x = 100$

Point: $(100, f(100)) = (100, \sqrt{100}) = (100, 10)$

$f'(x) = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

slope: $f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20}$

$y - 10 = \frac{1}{20}(x - 100)$

$y = \frac{1}{20}x - 5 + 10$

$y = \frac{1}{20}x + 5$

$\sqrt{x} \approx \frac{1}{20}x + 5$ for x near 100

$\sqrt{104} \approx \frac{1}{20}(104) + 5$

$\sqrt{104} \approx 10.2$ → from graphs, the exact value of $\sqrt{104}$ is less than our estimate

2. (4 points) There is one value of x for which the line tangent to the graph of $f(x) = x^4$ is parallel to the line tangent to the graph of $g(x) = x^3 + 5x + 2$. Determine this x-value using Newton's Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates x_2, x_3 and x_4. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

Two lines are parallel if they have the same slope.

Thus $f'(x) = g'(x)$

$$4x^3 = 3x^2 + 5$$

$$4x^3 - 3x^2 - 5 = 0$$

Let $w(x) = 4x^3 - 3x^2 - 5$ (thus $w'(x) = 12x^2 - 6x$)

and apply Newton's Method

$$x_{n+1} = x_n - \frac{w(x_n)}{w'(x_n)} = x_n - \frac{4x_n^3 - 3x_n^2 - 5}{12x_n^2 - 6x_n}$$

$x_1 = 1$ given as initial estimate

$$x_2 = 1 - \frac{4(1)^3 - 3(1)^2 - 5}{12(1)^2 - 6(1)} \approx 1.666666667$$

$$x_3 \approx 1.666666667 - \frac{4(1.666666667)^3 - 3(1.666666667)^2 - 5}{12(1.666666667)^2 - 6(1.666666667)}$$

$$x_3 \approx 1.444444444$$

$$x_4 \approx 1.444444444 - \frac{4(1.444444444)^3 - 3(1.444444444)^2 - 5}{12(1.444444444)^2 - 6(1.444444444)}$$

$$x_4 \approx 1.395843808$$