(circle your TA discussion section)

- **AD1**, TR 9:00-10:50, Nick Andersen
- **AD2**, TR 1:00-2:50, Sarah Loeb
- **ADA**, TR 8:00-8:50, Lisa Hickok
- **ADB**, TR 9:00-9:50, Sneha Chaubey
- **ADC**, TR 10:00-10:50, Sneha Chaubey
- **ADD**, TR 11:00-11:50, Tom Mahoney
- **ADE**, TR 12:00-12:50, Tom Mahoney
- **ADF**, TR 1:00-1:50, Lisa Hickok
- **ADG**, TR 2:00-2:50, Nathan Rehfuss
- **ADH**, TR 3:00-3:50, Nathan Rehfuss
- **ADJ**, TR 9:00-9:50, Dan Schultz
- **ADK**, TR 10:00-10:50, Dan Schultz
- **ADL**, TR 11:00-11:50, Derrek Yager
- **ADM**, TR 12:00-12:50, Derrek Yager
- **ADN**, TR 1:00-1:50, Ben Fulan
- **ADO**, TR 2:00-2:50, Ben Fulan
- **ADP**, TR 3:00-3:50, Mahmood Etedadi Aliabadi
- **ADQ**, TR 4:00-4:50, Mahmood Etedadi Aliabadi

- You may work with other MATH 220 students. However each student should write up solutions separately and independently – nobody should copy someone else’s work.

- You may use your notes or the textbook.

- Computers are not allowed on any problem. You may use a calculator only for basic arithmetic.

- The quiz should be submitted to Mr. Murphy at the beginning of your official lecture period on Friday, April 19th.

- There is a higher expectation for the quality of your work on a take-home quiz. Everything should be written logically and legibly with sufficient work to justify each answer. Blank copies of the quiz are available on the course home page.

- Be sure that the pages are nicely stapled – do not just fold the corners.

- **Note to TAs and Tutors** – you should not help students with these specific problems or go over solutions until after 5pm Friday.
1. (6 points) Use the techniques of linear approximation to estimate the following quantities. Write each answer in decimal form. No technology is allowed on this problem.

(a) $\ln(0.9)$

\[
y = x - 1
\]

we find the line tangent to $f(x) = \ln x$ at $x = 1$

Point: $(0, f(0)) = (0, \ln(1)) = (0, 0)$

$f'(x) = \frac{1}{x}$

slope: $f'(1) = \frac{1}{1} = 1$

\[y - 0 = 1 \cdot (x - 1)
\]

\[y = x - 1
\]

$\ln(0.9) \approx 0.9 - 1$  

$\ln(0.9) \approx -0.1$

From graphs, the exact value of $\ln(0.9)$ is less than our estimate.

(b) $\sqrt{104}$

\[
y = \frac{1}{50}x + 5
\]

we find the line tangent to $f(x) = \sqrt{x}$ at $x = 100$

Point: $(100, f(100)) = (100, \sqrt{100}) = (100, 10)$

$f'(x) = \frac{1}{2\sqrt{x}}$

slope: $f'(100) = \frac{1}{2\times10} = \frac{1}{20}$

\[y - 10 = \frac{1}{20} (x - 100)
\]

\[y = \frac{1}{20}x - 5 + 10
\]

\[y = \frac{1}{20}x + 5
\]

$\sqrt{104} \approx \frac{1}{50}(104) + 5$

$\sqrt{104} \approx 10.2$

From graphs, the exact value of $\sqrt{104}$ is less than our estimate.
2. (4 points) There is one value of $x$ for which the line tangent to the graph of $f(x) = x^4$ is parallel to the line tangent to the graph of $g(x) = x^3 + 5x + 2$. Determine this $x$-value using Newton’s Method with an initial estimate of $x_1 = 1$. You should use this method 3 times in order to obtain estimates $x_2$, $x_3$, and $x_4$. You are only allowed to use technology for basic arithmetic. Use at least 5 decimal places in each estimate.

Two lines are parallel if they have the same slope. Thus $f'(x) = g'(x)$

$4x^3 = 3x^2 + 5$

$4x^3 - 3x^2 - 5 = 0$

Let $w(x) = 4x^3 - 3x^2 - 5$ (thus $w'(x) = 12x^2 - 6x$)

and apply Newton’s Method

$x_{n+1} = x_n - \frac{w(x_n)}{w'(x_n)} = x_n - \frac{4x_n^3 - 3x_n^2 - 5}{12x_n^2 - 6x_n}$

$x_1 = 1$ given as initial estimate

$x_2 = 1 - \frac{4(1)^3 - 3(1)^2 - 5}{12(1)^2 - 6(1)} \approx 1.66666667$

$x_3 \approx 1.66666667 - \frac{4(1.66666667)^3 - 3(1.66666667)^2 - 5}{12(1.66666667)^2 - 6(1.66666667)}$

$x_3 \approx 1.44444444$

$x_4 \approx 1.44444444 - \frac{4(1.44444444)^3 - 3(1.44444444)^2 - 5}{12(1.44444444)^2 - 6(1.44444444)}$

$x_4 \approx 1.385843808$