

Name

SOLUTIONS

• You have 15 minutes

• No calculators

• Show sufficient work

1. (3 points) Suppose that $f(x) = 1 + \sqrt{3x-2}$ and $g(x) = 4 + \sqrt{10-2x}$. What is the domain of the composite function $(g \circ f)(x)$?

$$(g \circ f)(x) = g(f(x))$$

$$= g(1 + \sqrt{3x-2})$$

$$= 4 + \sqrt{10 - 2(1 + \sqrt{3x-2})}$$

$$= 4 + \sqrt{8 - 2\sqrt{3x-2}}$$

$$3x-2 \geq 0 \Rightarrow x \geq \frac{2}{3}$$

$$8 - 2\sqrt{3x-2} \geq 0$$

$$8 \geq 2\sqrt{3x-2}$$

$$4 \geq \sqrt{3x-2}$$

$$16 \geq 3x-2$$

$$18 \geq 3x$$

$$6 \geq x$$

$$x \geq \frac{2}{3} \text{ and } x \leq 6$$

$$\frac{2}{3} \leq x \leq 6$$

$$\text{domain}(g \circ f): \left[\frac{2}{3}, 6 \right]$$

2. (1 point) Evaluate the quantity $\csc(-\pi/6)$.

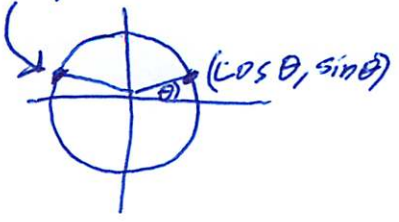
$$\csc\left(-\frac{\pi}{6}\right) = \frac{1}{\sin\left(-\frac{\pi}{6}\right)} = \frac{1}{-1/2} = -2$$

3. (3 points) Given an acute angle θ for which $\cos \theta = 1/3$, evaluate the following product.

$$\begin{aligned} \sin(\pi - \theta) \cos\left(\frac{\pi}{2} - \theta\right) &= \sin \theta \cdot \sin \theta \\ &= \sin^2 \theta \\ &= 1 - \cos^2 \theta \\ &= 1 - \left(\frac{1}{3}\right)^2 \\ &= \frac{8}{9} \end{aligned}$$

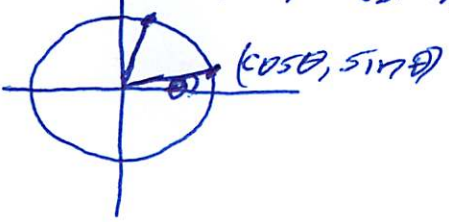
unit circles

$$(\cos(\pi - \theta), \sin(\pi - \theta))$$



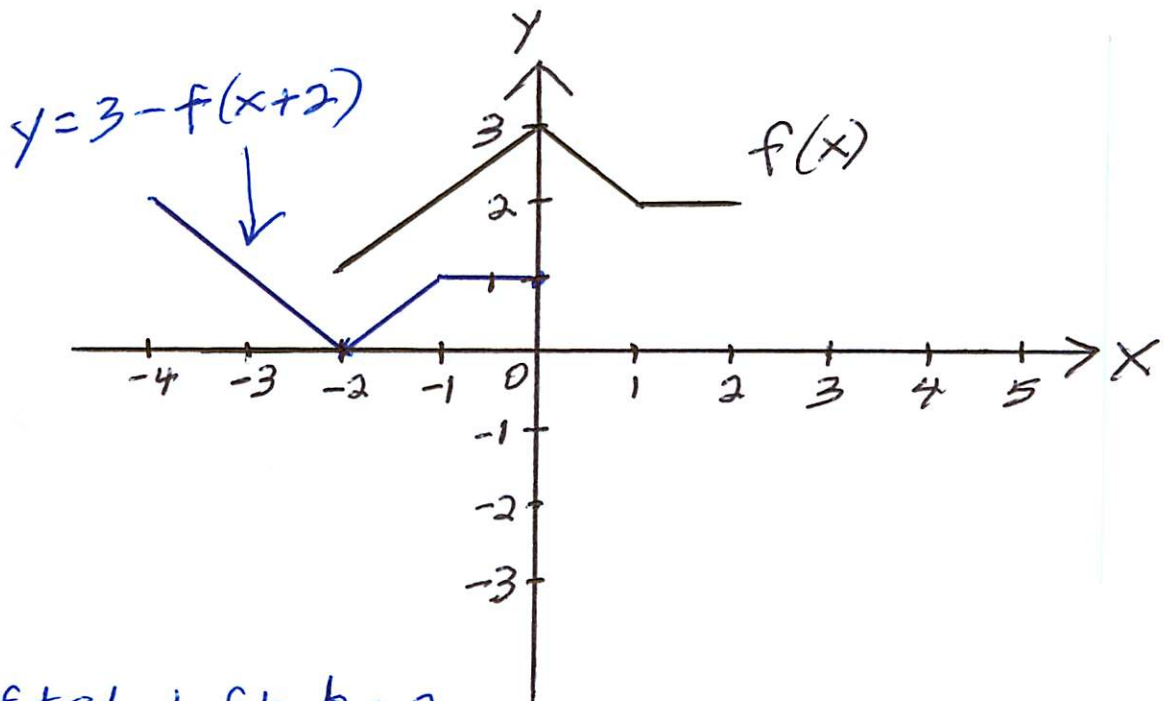
$$\sin(\pi - \theta) = \sin \theta$$

$$(\cos(\frac{\pi}{2} - \theta), \sin(\frac{\pi}{2} - \theta))$$



$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin \theta$$

4. (3 points) The graph of $y = f(x)$ is given. Draw the graph of $y = 3 - f(x + 2)$.



we shifted left by 2 units,
then flipped over the x -axis,
and finally shifted up by 3 units.