Name ___________________________________________________________

• No calculators allowed.
• Show sufficient work to justify each answer.
• You have 15 minutes for this quiz.

1. Find the indefinite integral.

(a) (2 pts) \( \int \left( \frac{2}{x^2} + \sin x - 1 \right) dx \)

\[ \int \left( \frac{2}{x^2} + \sin x - 1 \right) \, dx = -2x^{-1} - \cos x - x + C. \]

(b) (1 pt) \( \int \frac{2 + x^2}{1 + x^2} \, dx \)

\[ \int \frac{2 + x^2}{1 + x^2} \, dx = \int \frac{1 + (1 + x^2)}{1 + x^2} \, dx = \int \frac{dx}{1 + x^2} + \int dx = \arctan x + x + C. \]

2. (2 pts) A particle moves along a line with velocity function \( v(t) = t^2 - t \), where \( v \) is measured in meters per second. Find the distance traveled by the particle during the time interval \([1, 5]\).

\[ \text{Solution.} \text{ Note that on the domain } [1, 5] \text{ we have } |v(t)| = v(t), \text{ i.e., the velocity and the speed have the same sign. Thus the distance traveled by the particle during this time is} \]

\[ \int_1^5 |v(t)| \, dt = \int_1^5 v(t) \, dt = \int_1^5 (t^2 - t) \, dt = \left[ \frac{1}{3}t^3 - \frac{1}{2}t^2 \right]_1^5 = \frac{88}{3}. \]

3. Evaluate the definite integral.

(a) (2 pts) \( \int_0^1 \sqrt{1 - x^2} \, dx \). (Hint: Draw the graph of \( y = \sqrt{1 - x^2} \))
Solution. Note that the graph of the function \( y = \sqrt{1 - x^2} \) is the upper semi-circle of the circle of radius 1 centered at the origin. Thus the integral \( \int_0^1 \sqrt{1 - x^2} \, dx \) is the area of half of the upper semi-circle of this circle. In other words, this integral is the area of the circle restricted to the first quadrant. Thus

\[
\int_0^1 \sqrt{1 - x^2} \, dx = \frac{1}{4} \cdot \pi \cdot 1^2 = \frac{\pi}{4}.
\]

(b) (2pts) \( \int_{-\pi}^{\pi} \sin x \, dx \)

Solution. Recall that \( \sin x \) is an odd function, i.e., \( \sin(-x) = -\sin x \). This implies that

\[
\int_{-\pi}^{\pi} \sin x \, dx = -\int_{-\pi}^{0} \sin x \, dx.
\]

Thus

\[
\int_{-\pi}^{\pi} \sin x \, dx = \int_{-\pi}^{0} \sin x \, dx + \int_{0}^{\pi} \sin x \, dx = \int_{-\pi}^{0} \sin x \, dx - \int_{-\pi}^{0} \sin x \, dx = 0.
\]

(c) (1pts) \( \int_{0}^{4} \frac{1}{2x} \, dx \)

Solution. The function \( f(x) = \frac{1}{2x} \) is not continuous at 0. Hence the Fundamental Theorem of Calculus does not apply. The integral does not exist.