Name: [Key]

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (1 point) Given that \( \int_{5}^{8} e^{\sin^2 x} \ln x \, dx = 3698.88 \), and \( \int_{6}^{8} e^{\sin^2 x} \ln x \, dx = 3516.34 \), find \( \int_{5}^{8} e^{\sin^2 v \ln v} \, dv \)

\[
\int_{5}^{8} e^{\sin^2 v \ln v} \, dv = \int_{5}^{6} e^{\sin^2 v \ln v} \, dv + \int_{6}^{8} e^{\sin^2 v \ln v} \, dv
\]

\[
\int_{6}^{8} e^{\sin^2 v \ln v} \, dv = \int_{5}^{8} e^{\sin^2 x \ln x} \, dx - \int_{5}^{8} e^{\sin^2 x \ln x} \, dx = 3698.88 - 3516.34 = 182.54
\]

2. (1 point) If \( f(1) = 7 \), \( f' \) is continuous and \( \int_{1}^{8} f'(x) \, dx = 20 \), what is \( f(8) \)?

By FTC2, \( \int_{1}^{8} f'(x) \, dx = f(8) - f(1) \)

So \( 20 = f(8) - 7 \)

\( f(8) = 27 \)

3. (2 points) Fill in the missing information to show that the given definite integral can be expressed as the limit of a Riemann sum. The only variables appearing in your limit should be \( n \) and \( k \). You do not need to evaluate this limit.

\[
\int_{2}^{5} \frac{1}{\ln x + x^2} \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} \left[ \frac{1}{\ln x + x^2} \right]
\]

\( \Delta x = \frac{5 - 2}{n} = \frac{3}{n} \)

Using right endpoints, \( x_k = 2 + k \Delta x = 2 + \frac{3k}{n} \)

\[
\int_{2}^{5} \frac{1}{\ln x + x^2} \, dx = \lim_{n \to \infty} \sum_{k=1}^{n} f(x_k) \Delta x
\]

\[
= \lim_{n \to \infty} \sum_{k=1}^{n} f\left(2 + \frac{3k}{n}\right) \cdot \frac{3}{n}
\]

\[
= \lim_{n \to \infty} \sum_{k=1}^{n} \frac{1}{\ln\left(2 + \frac{3k}{n}\right) + \left(2 + \frac{3k}{n}\right)^2} \cdot \frac{3}{n}
\]
4. (3 points) Evaluate the following indefinite integral.

\[ \int \tan x \csc x \sec x \, dx \]

\[ \int \tan x \csc x \sec x \, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x} \cdot \frac{1}{\cos x} \, dx \]

\[ = \int \frac{1}{\cos^2 x} \, dx \]

\[ = \int \sec^2 x \, dx \]

\[ = \tan x + C \]

5. (3 points) At 9:00AM, Sue starts collecting donations for her charity. She collects money at a rate of \(6t + 10\) dollars per hour, where \(t\) denotes the number of hours since 9:00AM. What is the total amount of money that Sue collects between 11:00AM and 1:00PM? Simplify your answer as much as possible without the use of a calculator.

using net change thm

\[
\text{net}\$\text{\ collected}\quad \text{from } t=2 \ \text{to}\ t=4 = \int_{2}^{4} 6t+10 \, dt
\]

\[= \left. \frac{6t^2}{2} + 10t \right|_{2}^{4} \]

\[= \frac{6(4)^2}{2} + 10(4) - \left( \frac{6(2)^2}{2} + 10(2) \right) \]

\[= (48 + 40) - (12 + 20) \]

\[= 88 - 32 \]

\[= \$56 \]