You have 20 minutes for this quiz – no calculators allowed.

1. (2 points each) Evaluate the following integrals.

   (a) \[ \int_1^0 x^6 \, dx = \frac{x^{e+1}}{e+1} \bigg|_1^0 = \frac{0^{e+1}}{e+1} - \frac{1^{e+1}}{e+1} = -\frac{1}{e+1} \]

   (b) \[ \int \frac{1}{\csc x} \, dx = \int \sin x \, dx = -\cos x + C \]

2. (3 points) Evaluate the following limit. Be sure to use proper notation throughout your evaluation of this limit.

   \[ \lim_{n \to \infty} \sum_{k=1}^{n} \left( \frac{6k}{n^2} + \frac{-5}{n^2} \right) = \lim_{n \to \infty} \left( \sum_{k=1}^{n} \frac{6k}{n^2} + \sum_{k=1}^{n} \frac{-5}{n^2} \right) \]

   \[ = \lim_{n \to \infty} \left( \frac{6}{n^2} \cdot \frac{n(n+1)}{2} - \frac{5}{n} \right) \]

   \[ = \lim_{n \to \infty} \left( 3 \left(1+\frac{1}{n}\right) - \frac{5}{n} \right) \]

   \[ = 3(1) - 0 = 3 \]
3. (3 points) Suppose \( f \) is integrable on the interval \([5, 12]\). Given the following definite integrals,

\[
\int_{5}^{7} f(x) \, dx = 10 \\
\int_{8}^{12} f(x) \, dx = 5 \\
\int_{5}^{12} f(x) \, dx = 3
\]

what is the value of

(a) \( \int_{7}^{5} f(x) \, dx = \int_{7}^{12} f(x) \, dx - \int_{8}^{12} f(x) \, dx - \int_{5}^{7} f(x) \, dx \)

\[= 3 - 10 - 5 = \boxed{-12} \]

(b) \( \int_{7}^{12} f(x) \, dx = \int_{7}^{5} f(x) \, dx + \int_{8}^{12} f(x) \, dx = -12 + 5 = \boxed{-7} \)

(b) \( \int_{12}^{8} f(x) \, dx = - \int_{8}^{12} f(x) \, dx = \boxed{-5} \)