No calculators allowed.

Show sufficient work to justify each answer.

You have 15 minutes for this quiz.

1. (3 points) For the given function, use logarithmic differentiation to find a formula for \( \frac{dy}{dx} \) written in terms of \( x \).

\[ y = \cos(3x)^{7x^5} \]

First, take the natural log of both sides:

\[ \ln y = \ln(\cos(3x)^{7x^5}) \]

Now simplify using properties of log:

\[ \ln y = 7x^5 \ln(\cos(3x)) \]

Now do implicit differentiation:

\[ \frac{d}{dx}(\ln y) = \frac{d}{dx}(7x^5 \ln(\cos(3x))) \rightarrow \frac{y'}{y} = 35x^4 \cdot \ln(\cos(3x)) + (\ln(\cos(3x)))'7x^5 \]

\[ \frac{y'}{y} = 35x^4 \cdot \ln(\cos(3x)) + (\frac{1}{\cos(3x)})(3x^5)7x^5 \rightarrow \frac{y'}{y} = 35x^4 \ln(\cos(3x)) + \frac{1}{\cos(3x)}(-3x^5 \sin(3x))7x^5 \]

\[ \frac{y'}{y} = 35x^4 \ln(\cos(3x)) - \frac{21x^5 \sin(3x)}{\cos(3x)} \]

Using \( y = \cos(3x)^{7x^5} \) we get

\[ y' = (\cos(3x)^{7x^5})(35x^4 \ln(\cos(3x)) - \frac{21x^5 \sin(3x)}{\cos(3x)}) \]

2. (3 points) Given that \( \frac{dw}{dr} = 0.75w \) and \( w(4) = 4 \), find a formula for \( w \) as a function of \( r \).

The function that satisfies the differential equation \( \frac{dw}{dr} = 0.75w \) is the exponential:

\[ w(r) = Ce^{0.75r} \]

for some constant \( C \).

Now use the data \( w(4) = 4 \)

\[ w(4) = Ce^{0.75 \cdot 4} \]

\[ 4 = C e^{3} \]

\[ \frac{y}{e^3} = C \rightarrow w(t) = \frac{4}{e^3} e^{0.75t} = 4e^{0.75t - 3} \]
3. (4 points total) A ball is thrown straight up from an initial height of 5 feet above the ground. Until the ball hits the ground, the function \( h(t) = -10t^2 + 5t + 5 \) represents the ball's height in feet above ground level \( t \) seconds after it was thrown.

(a) (2 points) What is the velocity of the ball when it hits the ground?

The ball hits the ground when \( h(t) = 0 \), so solve

\[
-10t^2 + 5t + 5 = 0
\]

\( \Rightarrow \quad (-2t-1)(t-1) = 0 \quad \Rightarrow \quad t = 1 \) or \( t = -\frac{1}{2} \)

Since we are in the context of after the ball was thrown, \( t > 0 \).

Hence \( t = 1 \) not \( \frac{1}{2} \), so we want the velocity at time \( t = 1 \).

Velocity \( v = h'(t) = -20t + 5 \) \( \Rightarrow \quad h'(1) = -20 + 5 = -15 \) ft/s.

(b) (2 points) What is the maximum height of the ball?

Max height of ball is when \( h'(t) = 0 \),

\( 0 = h''(t) = -20 \). \( \Rightarrow \quad t = \frac{1}{4} \). So we want the

height at time \( t = \frac{1}{4} \)

\[
h\left(\frac{1}{4}\right) = -10\left(\frac{1}{4}\right)^2 + \frac{5}{4} + 5 = \frac{-10}{16} + \frac{5}{4} + 5
\]

\[
= \frac{-10 + 20 + 80}{16} = \frac{90}{16} = \frac{45}{8}
\]