QUIZ 6 BD6
SOLUTIONS

1. Start with taking logarithms of both sides:
   \[ \ln f(x) = \ln x^x. \]
   Then use properties of logarithms:
   \[ \ln f(x) = x \ln x. \]
   Now differentiate both sides:
   \[ \frac{1}{f(x)} f'(x) = \ln x + x \frac{1}{x} = \ln x + 1 \]
   Multiplying both sides by \( f(x) \) we are done
   \[ f'(x) = x^x (\ln x + 1). \]

2. We use the chain rule:
   \[ f'(x) = \frac{1}{\ln x} \cdot \frac{1}{x} = \frac{1}{x \ln x} \]
   To find \( f'' \) it is useful to write \( f' \) as
   \[ f'(x) = x^{-1} (\ln x)^{-1}. \]
   Now we use the chain rule together with the product rule:
   \[ f''(x) = -x^{-2} (\ln x)^{-1} + x^{-1} \cdot - (\ln x)^{-2} \frac{1}{x} = - \frac{1}{x^2 \ln x} - \frac{1}{x^2 (\ln x)^2}. \]