• No calculators allowed.
• Show sufficient work to justify each answer.
• You have 15 minutes for this quiz.

1. (3 pts) Use logarithmic differentiation to find $y'$ for $y = (\sin x)^{\ln x}$.

   $$\ln(y) = \ln((\sin x)^{\ln x}) = \ln x \cdot \ln(\sin x)$$

   $$\frac{1}{y} \cdot y' = \frac{1}{x} \ln(\sin x) + \ln(x) \cdot \frac{1}{\sin x} \cdot \cos x$$

   $$y' = y \left( \frac{\ln(\sin x)}{x} + \frac{\ln(x)\cos x}{\sin x} \right)$$

   $$y' = (\sin x)^{\ln x} \left( \frac{\ln(\sin x)}{x} + \frac{\ln(x)\cos x}{\sin x} \right)$$

2. The half-life of radium-226 is 1590 years.

   (a) (2 pts) A sample of radium-226 has a mass of 100mg. Find a formula for the mass of the sample that remains after $t$ years.

   Let $y(t) = \text{Mass after } t \text{ years} \Rightarrow$

   $$y(t) = y(0) e^{kt} \quad y(0) = 100 \Rightarrow y(t) = 100e^{kt}$$

   $$\frac{100}{2} = 100 e^{1590k} \quad \Leftrightarrow \quad \frac{1}{2} = e^{1590k} \quad \Leftrightarrow \quad -\ln(2) = 1590k$$

   $$\Rightarrow \quad \frac{-\ln(2)}{1590} = k \quad \text{so} \quad y(t) = 100 e^{\frac{-\ln(2)}{1590} t}$$
(b) (2 pts) Find the mass after 1000 years correct to the nearest milligram.

\[ \text{Find } y(1000) \]

\[ y(1000) = 100 e^{\frac{-\ln(2) \cdot 1000}{1540}} \]

3. A particle moves on a vertical line so that its coordinate at time \( t \) is \( y = t^3 - 12t + 3, t \geq 0 \).

(a) (1 pt) Find the velocity function.

(b) (1 pt) Find the acceleration function

(a) \( v = y' = 3t^2 - 12t \)

(b) \( a = y'' = 6t \)

(c) (1 pt) Find the velocity of the function at time \( t = 1 \).

\[ v(1) = 3(1)^2 - 12 = 3 - 12 = -9 \]