No calculators allowed.

Show sufficient work to justify each answer.

You have 15 minutes for this quiz.

1. The position function of a particle moving along a wire is given by \( s(t) = t^3 - 6t^2 \), where \( s \) is in meters and \( t \geq 0 \) is in seconds.

   (a) (2 pts) Find the acceleration of the particle 3 seconds after it begins moving.

   \[
   v(t) = s'(t) = 3t^2 - 12t
   \]

   \[
   a(t) = s''(t) = 6t - 12
   \]

   \[
   a(3) = 6(3) - 12 = 18 - 12 = 6 \text{ m/s}^2
   \]

   \[
   a(3) = 6 \text{ m/s}^2
   \]

   (b) (2 pts) What is the position of the particle when it reverses direction?

   When the particle reverses direction, its instantaneous velocity is 0.

   \[
   v(t) = 3t^2 - 12t = 0
   \]

   \[
   3t(t - 4) = 0
   \]

   \[
   t = 0 \text{ or } t = 4
   \]

   \[
   \uparrow
   \]

   This can be ignored, since we never have \( t < 0 \).

   When \( t = 4 \),

   \[
   s(4) = 4^3 - 6(4^2) = 64 - 6(16) = 64 - 96 = -32.
   \]

   The particle is at position \(-32 \text{ m}\) when it reverses direction.
2. (3 pts) Given that \( r = (3t^2 + 2)^{4t^3} \), find a formula for \( \frac{dr}{dt} \). Your formula must be written in terms of \( t \).

\[
\frac{dr}{dt} = \frac{d}{dt} \left( (3t^2 + 2)^{4t^3} \right) \\
= 4t^3 \ln(3t^2 + 2) (3t^2 + 2)^{4t^3} \\
= 12t^2 \ln(3t^2 + 2) + 4t^3 \cdot \frac{1}{3t^2 + 2} \cdot 6t \\
= 12t^2 \ln(3t^2 + 2) + \frac{24t^4}{3t^2 + 2} \\
\frac{dr}{dt} = (3t^2 + 2)^{4t^3} \left( 12t^2 \ln(3t^2 + 2) + \frac{24t^4}{3t^2 + 2} \right)
\]

3. (3 pts) The decay of a radioactive substance is described by the equation \( \frac{dA}{dt} = -0.04A \), where \( A \) is the amount of the substance and \( t \) is the time in weeks. If you start with 100g of this substance, how long will it take to decay to 25g?

\[
\frac{dA}{dt} = -0.04A, \text{ so } A(t) = Ce^{-0.04t} \\
A(0) = 100g, \text{ so } 100 = Ce^{-0.04(0)} = C \cdot e^0 = C \cdot 1 = C \\
\text{Thus } A(t) = 100e^{-0.04t}.
\]

We want to find \( t \) such that \( A(t) = 25g \).

\[
25 = 100e^{-0.04t} \\
e^{-0.04t} = \frac{25}{100} = \frac{1}{4} \\
\ln(e^{-0.04t}) = \ln\left(\frac{1}{4}\right) \\
-0.04t = \ln\left(\frac{1}{4}\right) \\
t = \frac{\ln\left(\frac{1}{4}\right)}{-0.04} \text{ weeks}
\]

Note: \( \ln\left(\frac{1}{4}\right) = \ln(4^{-1}) = -1 \ln 4 \), so \( t = \frac{\ln 4}{0.04} \text{ weeks} \)

and \( -\ln 4 = -\ln(2^2) = -2\ln 2 \), so \( t = \frac{2\ln 2}{0.04} = \frac{2\ln 2}{0.02} \text{ weeks} \)

are equivalent answers.
Name: KEY

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. The position function of a particle moving along a wire is given by \( s(t) = t^3 - 3t^2 \), where \( s \) is in meters and \( t \geq 0 \) is in seconds.

   (a) (2 pts) Find the acceleration of the particle 3 seconds after it begins moving.
   
   \[
   \begin{align*}
   v(t) &= s'(t) = 3t^2 - 6t \\
   a(t) &= s''(t) = 6t - 6 \\
   a(3) &= 6(3) - 6 = 18 - 6 = 12 \\
   \underline{a(3) = 12 \text{m/s}^2}
   \end{align*}
   \]

   (b) (2 pts) What is the position of the particle when it reverses direction?

   When the particle reverses direction, its instantaneous velocity is 0.
   
   \[
   v(t) = 3t^2 - 6t = 0 \\
   3t(t - 2) = 0 \\
   t = 0 \text{ or } t = 2
   \]

   \( t = 0 \) can be ignored, since we never have \( t < 0 \).

   When \( t = 2 \),
   
   \[
   s(2) = 2^3 - 3(2^2) = 8 - 12 = -4.
   \]

   The particle is at position \(-4 \text{m}\) when it reverses direction.
2. (3 pts) Given that \( r = (2t^4 + 4)^{3t^2} \), find a formula for \( \frac{dr}{dt} \). Your formula must be written in terms of \( t \).

\[
\frac{dr}{dt} = 6t \ln(2t^4 + 4) + \frac{1}{2t^3 + 4} \cdot 8t^3
\]

\[
\frac{dr}{dt} = (2t^4 + 4)^{3t^2} \left( 6t \ln(2t^4 + 4) + \frac{24t^5}{2t^4 + 4} \right)
\]

3. (3 pts) The decay of a radioactive substance is described by the equation \( \frac{dA}{dt} = -0.07A \), where \( A \) is the amount of the substance and \( t \) is the time in weeks. If you start with 50g of this substance, how long will it take to decay to 25g?

\[
\frac{dA}{dt} = -0.07A, \quad \text{so} \quad A(t) = Ce^{-0.07t}
\]

\[A(0) = 50g, \quad \text{so} \quad 50 = Ce^{-0.07(0)} = C \cdot e^0 = C \cdot 1 = C.
\]

Thus \( A(t) = 50e^{-0.07t} \).

We want to find \( t \) such that \( A(t) = 25g \).

\[25 = 50e^{-0.07t}
\]

\[-0.07t = \ln(\frac{25}{50}) = \ln(\frac{1}{2})\]

\[-0.07t = \frac{\ln(\frac{1}{2})}{-0.07} \text{ weeks}\]

Note: \( \ln(\frac{1}{2}) = \ln(2^{-1}) = -1 \ln 2 \), so \( t = \frac{\ln 2}{0.07} \text{ weeks} \) is an equivalent answer.