Math 220 (section AD4)          Quiz 6          Spring 2012

Name

• No calculators allowed.

• Show sufficient work to justify each answer.

• You have 15 minutes for this quiz.

1. (4 points) Use logarithmic differentiation to find \( \frac{dy}{dx} \). Your answer must be all in terms of \( x \).

\[ y = (\csc x)^x \]

\[
\ln(y) = \ln((\csc x)^x) \\
\ln(y) = x \ln(\csc x) \\
\frac{1}{y} \frac{dy}{dx} = x \cdot \frac{1}{\csc x} \cdot -\csc x \cot x + 1 \cdot \ln(\csc x) \\
\frac{dy}{dx} = y \left[ -\frac{x \csc x \cot x}{\csc x} + \ln(\csc x) \right] \\
\frac{dy}{dx} = (\csc x)^x \left( -x \cot x + \ln(\csc x) \right) 
\]
2. (3 points) A particle moves along the x-axis and its position at time $t$ is given by

$$s(t) = \frac{t^4}{12} - \frac{3t^3}{2} + 4t^2 + \frac{3t}{5} + 7$$

At what time(s) is the acceleration of the particle 0?

$$\alpha(t) = \frac{d^2s}{dt^2} = \frac{2t^2}{3} - \frac{18t}{2} + 8 = t^2 - 9t + 8$$

$$t^2 - 9t + 8 = 0$$

$$(t - 8)(t - 1) = 0$$

Acceleration 0 when $t = 1, t = 8$

3. (3 points) The half life of Megnesium is 11 years. What percentage of the original mass is left after 2 years. (It is ok to leave ln in your answer)

$$\frac{1}{2}m_0 = m_0 e^{-11K} \quad \text{(after 11 years } \frac{1}{2} \text{ is left)}$$

$$\frac{1}{2} = e^{-11K}$$

$$\ln(1/2) = 11K$$

$$\ln(1) - \ln(2) = 11K$$

$$-\ln(2) = 11K$$

$$K = -\frac{\ln(2)}{11} \quad \text{and this is the rate}$$

$$m = m_0 e^{-\frac{\ln(2)}{11} \cdot t} \quad \text{this is the formula for decay}$$

When $t = 2$, $m = m_0 e^{-\frac{\ln(2)}{11} \cdot 2}$ and plug in 2 years.