Name: Solutions

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (4 points) Compute $\frac{dy}{dx}$ given that $\ln(y) = x^2 + \sin(5y^2)$.

We treat $y$ as a function of $x$ (i.e., $y = f(x)$) and then differentiate the expression with respect to $x$. Recall $y' = \frac{dy}{dx}$.

\[
\frac{d}{dx} \ln(y) = \frac{d}{dx} (x^2 + \sin(5y^2))
\]

Recall the chain rule,

\[
\frac{d}{dx} \ln(y) = \frac{1}{y} y' \\
\frac{d}{dx} \sin(5y^2) = \cos(5y^2)(5y^2)' \\
= \cos(5y^2) 10y y'
\]

Now we solve for $y'$.

\[
y' = 2x + 10y y' \cos(5y^2)
\]

\[-10y^2 y' \cos(5y^2) + y' = 2x y
\]

\[y' (10y^2 \cos(5y^2) + 1) = 2xy
\]

\[y' = \frac{2xy}{-10y^2 \cos(5y^2) + 1} \left( = \frac{2x}{-10y \cos(5y^2) + \frac{1}{y}} \right)
\]
2. (3 points) Find the slope of the line tangent to the graph of the curve 

\[ y = \cos(x) \sin(x) \]

at the point \( \left( \frac{\pi}{4}, \frac{1}{2} \right) \).

First we find \( \frac{dy}{dx} = y' \). We use the product rule,

\[ y' = (\cos(x))' \sin(x) + \cos(x)(\sin(x))' \]

\[ = -\sin^2(x) + \cos^2(x) \]

Now we plug in the point \( (x, y) = \left( \frac{\pi}{4}, \frac{1}{2} \right) \).

\[ \frac{dy}{dx} \bigg|_{x=\frac{\pi}{4}} = -\sin^2\left(\frac{\pi}{4}\right) + \cos^2\left(\frac{\pi}{4}\right) \]

\[ = -\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 0 \]

So the slope at \( \left( \frac{\pi}{4}, \frac{1}{2} \right) \) is 0.

3. (3 points) Compute \( g'(t) \) where \( g(t) = (\tan^2(t) + 2t)^4 \).

First we apply the power rule of functions

\[ g'(t) = 4(tan^2(t) + 2t)^3(2\tan(t) + 2) \]

Use power rule again

\[ (\tan^2(t))' = 2\tan(t)(\sec^2(t)) \]

\[ (2t)' = 2 \]