1. Using Leibniz notation \( \frac{df}{dx} \), etc, find the derivatives of the following. Do not simplify.

(a) **(2 points)** \( f(x) = \frac{\sqrt{x} - 10x^5}{x^7} \)

First, simplify: \( f(x) = x^{1/2} - 10x^5 = x^{-13/2} - 10x^3 \) so

\[
\frac{df}{dx} = \frac{d}{dx} x^{-13/2} + \frac{d}{dx} (-10x^3) = -\frac{13}{2}x^{-15/2} + 20x^{-3}
\]

(b) **(2 points)** \( g(t) = 6t^6e^t + \pi^6e^t + \pi^6e^\pi \)

We need the product rule:

\[
\frac{dg}{dt} = \frac{d}{dt} (6t^6e^t) + \frac{d}{dt} (\pi^6e^t) + \frac{d}{dt} (\pi^6e^\pi) = 6t^6e^t + 6t^5e^t + \pi^6e^t + 0
\]

(c) **(2 points)** \( h(z) = \frac{z^6 + \sin(3)z - \ln(6)}{e^z - 18z} \)

We need the quotient rule:

\[
\frac{dh}{dz} = \frac{d}{dz} \left( \frac{z^6 + \sin(3)z - \ln(6)}{e^z - 18z} \right) = \frac{(6z^5 + \sin(3))(e^z - 18z) - (e^z - 18)(z^6 + \sin(3)z - \ln(6))}{(e^z - 18z)^2}
\]

2. **(4 points)** Let \( f(x) = \frac{1}{3}x^3 - x^2 + 3x + \frac{2}{3} \)

Find the equations of all lines which are both tangent to \( f \) and parallel to the line \( y = 2x + 8 \).

We want tangent lines, and the slope of a tangent line is always given by \( f'(x) \). So let’s first compute

\[
f'(x) = x^2 - 2x + 3
\]

Let’s say our tangent line goes through \( x = a \) (we’ll need to find \( a \) eventually!). It will then have the form

\[
y - f(a) = f'(a)(x - a)
\]

so for this line to be parallel to \( y = 2x + 8 \), we need the two lines to have the same slope, namely, \( f'(a) = 2 \). So we need to solve

\[
a^2 - 2a + 3 = 2
\]

for \( a \):

\[
a^2 - 2a + 3 = 2 \implies a^2 - 2a + 1 = 0 \implies (a - 1)^2 = 0 \implies a = 1
\]

This tells us that there is only one point, \( a = 1 \), at which the tangent line is parallel to \( y = 2x + 8 \). Our job now is to find the equation of this tangent line:

\[
f(1) = 1/3 - 1 + 3 + \frac{2}{3} = 3
\]

and we know \( f'(1) = 2 \) from the above, so our line is

\[
y - 3 = 2(x - 1)
\]