1. **(2 points)** Compute \( \sec(\tan^{-1}(3)) \)

Let \( \theta = \tan^{-1}(3) \). Thus, \( \tan(\theta) = 3 \). Since tangent = opp/adj, we can draw the following triangle:

\[
\begin{align*}
3 & \quad \theta \\
\sqrt{10} & \\
1 & \quad 1
\end{align*}
\]

It is also worth noting that since \( \tan(\theta) > 0 \), we know that \( 0 < \theta < \pi/2 \), so all the above lengths are indeed positive.

Since secant = hyp/adj, we conclude that

\[
\csc(\tan^{-1}(3)) = \csc(\theta) = \frac{\sqrt{10}}{1} = \sqrt{10}
\]

2. **(2 points)** Find the value of the number \( c \) so that the following function \( f \) is continuous everywhere.

\[
f(x) = \begin{cases} 
\frac{1 - x^2}{1 - x} & \text{for } x \neq 1 \\
c & \text{for } x = 1
\end{cases}
\]

We need to obtain \( \lim_{x \to 1} f(x) = f(1) \), that is:

\[
\lim_{x \to 1} \frac{1 - x^2}{1 - x} = c
\]

Plugging in 0 gets us 0/0. So:

\[
\lim_{x \to 1} \frac{1 - x^2}{1 - x} = \lim_{x \to 1} \frac{(1 - x)(1 + x)}{1 - x} = \lim_{x \to 1} 1 + x = 2
\]

Thus, \( c = 2 \).

3. Evaluate the following. On each, show your work and/or explain your reasoning.

(a) **(2 points)** \( \lim_{x \to 2} \frac{\ln(x)}{3x^2 - 12} \)

The numerator goes to \( \ln(2) \). The denominator goes to 0. In particular for \( x \) near 2 but less than 2, the numerator is near \( \ln(2) \) (no problems there!) and the denominator is a small negative number (near 0), so the fraction is a large negative number. Thus the limit is \( -\infty \).

(b) **(2 points)** \( \lim_{x \to 1} \frac{2 - 2x}{2 - \sqrt{x + 3}} \)
The numerator and denominator go to 0. Try multiplying by conjugates:

\[
\lim_{x \to 1} \frac{2 - 2x}{2 - \sqrt{x + 3}} \frac{2 + \sqrt{x + 3}}{2 + \sqrt{x + 3}} = \lim_{x \to 1} \frac{2(1 - x)(2 + \sqrt{x + 3})}{4 - (x + 3)} = \lim_{x \to 1} \frac{2(1 - x)(2 + \sqrt{x + 3})}{1 - x}
\]

\[
= \lim_{x \to 1} \frac{2(2 + \sqrt{x + 3})}{1 - x} = 2(2 + \sqrt{4}) = 8
\]

(c) (2 points) \( \lim_{x \to \infty} \frac{x^2 + 3x + 2}{4 - 5x^2 + 4x} \)

The numerator goes to \( \infty \) and the denominator goes to \( -\infty \). We use the standard trick:

\[
\lim_{x \to \infty} \frac{x^2 + 3x + 2}{4 - 5x^2 + 4x} \cdot \frac{1/x^2}{1/x^2} = \lim_{x \to \infty} \frac{1/x^2 + 2/x^2}{4/x^2 - 5 + 4/x} = \frac{-1}{5}
\]