SOLUTIONS TO QUIZ 2

1. a. Adding 1/4 to both sides we get

\[ y + \frac{1}{4} = x^2 - x + \frac{1}{4} = (x - \frac{1}{2})^2. \]

Since \( x \geq \frac{1}{2} \) we take square roots of both sides and have

\[ (y + \frac{1}{4})^{1/2} = x - \frac{1}{2} \]

(if we had \( x \leq \frac{1}{2} \) the right hand side would be \(-x + \frac{1}{2}\)). Then

\[ x = (y + 1/4)^{1/2} + 1/2. \]

Thus the inverse function is

\[ y = (x + 1/4)^{1/2} + 1/2. \]

b. Multiplying both sides by \( 1 + 2e^x \) we get

\[ y + 2ye^x = e^x. \]

Then we put \( e^x \) terms on one side

\[ y = e^x(-2y + 1). \]

Dividing both sides by \(-2y + 1\)

\[ e^x = \frac{y}{-2y + 1} \]

. Now we take logarithms of both sides

\[ x = \ln \frac{y}{-2y + 1} \]

So the inverse function is

\[ y = \ln \frac{x}{-2x + 1}. \]

2. a. Taking exponentials of both sides

\[ e^{\ln(x)} = e \]

\[ 1 \]
which gives
\[ \ln x = e. \]
Taking exponentials once more
\[ e^{\ln x} = e^e \]
from which
\[ x = e^e. \]

**b.** If \( C \leq 0 \) there can be no solution as left hand side is strictly positive. If \( C > 0 \) we proceed as follows: taking logarithms of both sides
\[ \ln(e^{ax}) = \ln(Ce^{bx}). \]
Then we have
\[ ax = \ln C + \ln e^{bx} = \ln C + bx. \]
Thus
\[ x(a - b) = \ln C \]
and
\[ x = \frac{\ln C}{a - b}. \]