Name ______ KEY ______

- No calculators allowed.
- Show sufficient work to justify each answer.
- You have 15 minutes for this quiz.

1. (3 pts) An exponential function has a $y$-intercept of 6 and passes through the point $(8, 24)$. Determine a formula for this function.

$y$-intercept = 6 \Rightarrow (0, 6)$ is on the graph of $f$

$f(x) = C a^x$

$(0, 6) \Rightarrow 6 = C a^0 = C$

so \hspace{1em} C = 6, \hspace{1em} and \hspace{1em} f(x) = 6a^x

$(8, 24) \Rightarrow 24 = 6a^8$

$a^8 = \frac{24}{6} = 4$

$a = 4^{\frac{1}{8}} = \left(2^2\right)^{\frac{1}{8}} = 2^{\frac{1}{4}}$

Thus

$f(x) = 6 \cdot \left(2^{\frac{1}{4}}\right)^x$

or equivalently

$f(x) = 6 \cdot 2^{x/4}$

or

$f(x) = 6 \cdot \left(\sqrt[4]{2}\right)^x$
2. (3 pts) Solve for $x$ in the equation below.

\[
\ln(x^2 + 2x - 14) = 0
\]

\[
e^{\ln(x^2 + 2x - 14)} = e^0
\]

\[
x^2 + 2x - 14 = 1
\]

\[
x^2 + 2x - 15 = 0
\]

\[
(x + 5)(x - 3) = 0
\]

\[
x = -5 \text{ or } x = 3
\]

Check $x$ is in domain:

\[
x = -5:
\]

\[
(-5)^2 + 2(-5) - 14 = 25 - 10 - 14 = 1 > 0
\]

\[
x = 3:
\]

\[
3^2 - 2(3) - 14 = 9 + 6 - 14 = 1 > 0
\]

So both values of $x$ are in the domain, and

\[
\boxed{x = -5 \text{ or } x = 3}
\]

3. (3 pts) Given that $g(x) = 3e^{x^2 + 4}$, find a formula for $g^{-1}(x)$.

\[
y = 3e^{x^2 + 4}
\]

\[
x = 3e^y^{x^2 + 4}
\]

\[
e^y = \frac{x}{3}
\]

\[
y^2 + 4 = \ln\left(\frac{x}{3}\right)
\]

\[
y^2 = \ln\left(\frac{x}{3}\right) - 4
\]

\[
y = \sqrt{\ln\left(\frac{x}{3}\right) - 4}
\]

\[
\boxed{g^{-1}(x) = \sqrt{\ln\left(\frac{x}{3}\right) - 4}}
\]

4. (1 pt) Suppose that $f$ is a one-to-one function which takes on the following values.

\[
f(-3) = 3, f(-2) = -4, f(-1) = 2, f(0) = -1/3, f(1) = 1/2, f(2) = -3, f(3) = 0
\]

What is the value of $f^{-1}(2)$?

\[
\boxed{f^{-1}(2) = -1}
\]
1. (3 pts) An exponential function has a y-intercept of 4 and passes through the point (8, 36). Determine a formula for this function.

$\text{y-intercept } = 4 \Rightarrow (0, 4) \text{ is on the graph of } f$

$f(x) = c a^x$

$(0, 4) \Rightarrow 4 = c a^0 = c,$

so $c = 4$, and we have $f(x) = 4a^x$

$(8, 36) \Rightarrow 36 = 4a^8$

$a^8 = \frac{36}{4} = 9$

$a = 9^{\frac{1}{8}} = (3^2)^{\frac{1}{8}} = 3^{\frac{1}{4}}$

Thus $f(x) = 4 \cdot (3^{\frac{1}{4}})^x$

or equivalently,

$f(x) = 4 \cdot 3^{\frac{x}{4}}$

or $f(x) = 4 \cdot (\sqrt[4]{3})^x$
2. (3 pts) Solve for $x$ in the equation below.

\[
\ln(x^2 - 8x + 16) = 0
\]

$e^{\ln(x^2 - 8x + 16)} = e^0$

$x^2 - 8x + 16 = 1$

$x^2 - 8x + 15 = 0$

$(x - 5)(x - 3) = 0$

$x = 5$ or $x = 3$

Check $x$ is in domain:

$x = 5$:

$5^2 - 8(5) + 16 = 25 - 40 + 16 = 1 > 0$  \(\checkmark\)

$x = 3$:

$3^2 - 8(3) + 16 = 9 - 24 + 16 = 1 > 0$  \(\checkmark\)

So both values of $x$ are in the domain, and

\[x = 5 \text{ or } x = 3\]

3. (3 pts) Given that $g(x) = 3e^{\sqrt{x} + 5}$, find a formula for $g^{-1}(x)$.

\[
y = 3e^{\sqrt{x} + 5}
\]

\[
x = 3e^{\sqrt{y} + 5}
\]

\[
e^{\sqrt{y} + 5} = \frac{x}{3}
\]

\[
\sqrt{y} + 5 = \ln\left(\frac{x}{3}\right)
\]

\[
\sqrt{y} = \ln\left(\frac{x}{3}\right) - 5
\]

\[
y = \left(\ln\left(\frac{x}{3}\right) - 5\right)^3
\]

So

\[g^{-1}(x) = \left(\ln\left(\frac{x}{3}\right) - 5\right)^3\]

4. (1 pt) Suppose that $f$ is a one-to-one function which takes on the following values.

\[f(-3) = 3, f(-2) = -4, f(-1) = 2, f(0) = -1/3, f(1) = 1/2, f(2) = -3, f(3) = 0\]

What is the value of $f^{-1}(3)$?

\[f^{-1}(3) = -3\]